Crisis induced intermittency in a bailout embedding map

We study the dynamics of inertial particles in two dimensional incompressible flows modelled by $4-d$ dissipative bailout embedding maps of the base flow which is represented by $2-d$ area preserving maps. An attractor widening cum merging crisis is seen at a point in the parameter space of the aerosols. Crisis induced intermittency is seen in the vicinity of this point. The distribution of the characteristic times follows a power law as a function of the dissipation parameter.

INTRODUCTION

The dynamics of inertial particles in $2-d$ incompressible fluids is modelled by the $4-d$ embedding map obtained from the minimal version of the Maxey-Riley equations[1, 2]. The embedding map is given by,

$$\begin{aligned} \mathbf{x}_{n+1} &= \mathbf{M}(\mathbf{x}_n) + \delta_n \\ \delta_{n+1} &= e^{-\gamma}[\alpha \mathbf{x}_{n+1} - \mathbf{M}(\mathbf{x}_n)]. \end{aligned} \quad (1)$$

This system has two parameters - γ, the dissipation parameter and α, the mass ratio parameter. The particles which are heavier than the fluid are called aerosols ($\alpha < 1$) and particles which are lighter than the fluid are called bubbles ($\alpha > 1$). The variable δ defines the detachment of the particle from the local fluid parcel. In our study we use the standard map, ($\mathbf{x}_{n+1} = \mathbf{x}_n + \mathbf{y}_{n+1}; \mathbf{y}_{n+1} = \mathbf{y}_n + (K/2\pi)\sin(2\pi \mathbf{x}_n)$) as the base map modelling the fluid dynamics.

An attractor widening cum merging crisis is seen for a special regime in the aerosol case. In the vicinity of crisis, as the dissipation parameter γ is varied through γ_c (the critical parameter value), two period-10 attractors in the phase space merge and widen to form a single large attractor (Fig. 1). The maximum Lyapunov exponent shows a sudden jump (see Fig. 2) signifying the crisis[3]. Crisis induced intermittency is seen in the neigbourhood of the γ_c. The distribution of the average characteristic time τ in the neighbourhood of γ_c was seen to follow a power law [4], $\tau \sim (\gamma_c - \gamma)^{\beta}$ (see Fig. 3). The power law exponent was found to be $\beta = -0.35$.

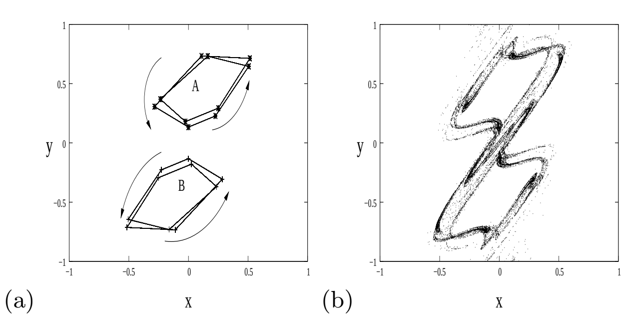

FIG. 1: The phase space plots for (a) the pre-crisis attractor with two period-10 orbits; $\gamma = 0.41$ (b)the post-crisis attractor; $\gamma = 0.40$.

CONCLUSION

The results our study can have implications for the dynamics of impurities in diverse application contexts, e.g. that of the dispersion of pollutants in the atmosphere, flows with suspended microstructures, coagulation of material particles in flows and catalytic chemical reactions.

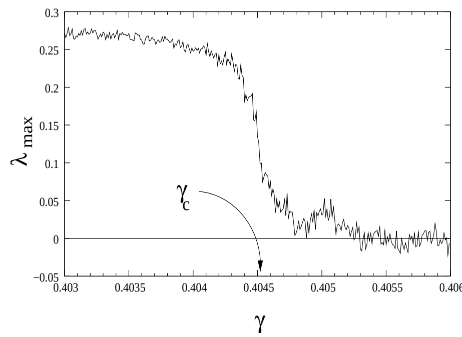

FIG. 2: Plot of the maximum Lyapunov exponent versus the dissipation parameter γ in the vicinity of crisis, shows a sudden jump.

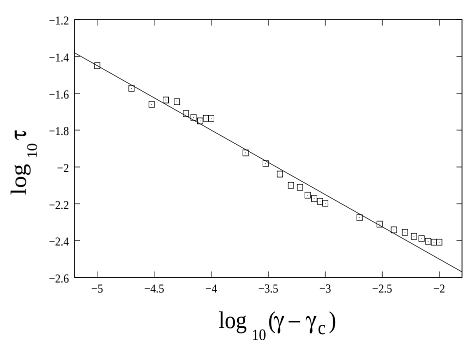

FIG. 3: The log-log plot of the characteristic time τ versus $\gamma_c - \gamma$ with the slope $\beta = -0.35 \pm 0.065087$.

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