

Abstract for Period Three implies Ergodicity.

The title of this paper is a reference to the famous paper by York and Li, "Period three implies chaos." It was here where the term 'chaos' was first used as a mathematical characterization. Our goal is to establish the connection between Ergodicity and Chaos. York and Li proved that given a continuous dynamical system, the existence of a Period 3 limit cycle implies that the system is chaotic. We wish to take the theorem a step forward to prove that under certain conditions, Period 3 implies Ergodicity as well.

Specifically given a dynamical system (X, F, T, m) with the following conditions:

- 1: T is continuous
- 2: There exists a Period three limit cycle.
- 3: m is area like.
- 4: For any closed subset of X : A , there exists a nonempty open set, B , with the property $m(B-A) = 0$

Then (X, F, T, m) is Ergodic.

Note: The notion of an area-like measure was developed by Oxtoby and Ullam in 1941, and is a way of cornering 'nice' properties into a measure.

References

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- [Walters] P. Walters, Introduction to Ergodic Theory. New York: Springer-Verlag, 2000.