Local stable manifold theorem for fractional systems

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Fractional Calculus is receiving considerable attention and has been one of the active areas of research in recent three decades or so. An important aspect of this subject which was initiated by Grigerinko and Grigerinko [1] involves the study of dynamics of fractional systems. Further extensive simulation work was carried out by various researchers [2]. Various aspects of fractional dynamical systems have been investigated which includes stability analysis[3, 4, 5], chaos, synchronization, delay [6] and so on. Powerful numerical methods have been developed to solve fractional differential equations(FDEs) [7] and analytical methods such as ADM [8, 9], NIM [10] have been employed extensively in the literature. Though much numerical work has been carried out to understand dynamical structure, analytical results obtained are very few. The main obstacle in generalizing the results to fractional ordered systems is that the solutions fail to satisfy semi-group property and hence do not generate *flow* in the traditional sense.

In pursuance to this topic, in the present paper, we develop asymptotics and some relevant properties of Mittag-Leffler function with matrix argument [11]. Further we generalize the local stable manifold theorem for fractional order systems, using Mittag-Leffler functions of matrix arguments and its properties. We illustrate this result with an example and plot corresponding local stable manifold.

Full paper published in A. Deshpande and V. Daftardar-Gejji, Local stable manifold theorem for fractional systems, *Nonlinear Dynamics*, **83 no 4**, 2016, pp. 2435-2452.

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