Modeling and Stability of a Smart Grid as a Coupled Oscillator

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This work is motivated by growing interest of modeling a smart grid as a coupled oscillator [1]. A Scale Free Network (SFN) is considered as a coupled oscillator to represent a smart grid [2] [3]. For illustration, a SFN with five nodes is shown in Fig. 1. The SFN is modeled by using Kuramoto first order eqn. 1,

$$\dot{\theta}_i = \omega_i + K\sum_{j=1}^{N} A_{ij} sin(\theta_j - \theta_i) \tag{1}$$

where θ_i is the phase of i^{th} node oscillator, N is the total number of nodes, K is the coupling strength, A_{ij} is the adjacency matrix where $A_{ij} = 1$ if nodes i and j are connected and $A_{ij} = 0$ if nodes i and j have no connection, and ω_i is the angular frequency of the i^{th} node [4]. Here, we investigate nonlinearity in the network. To study the natural behavior of the system, we set $K = 0.3$, ω_i is drawn from uniform distribution of zero mean and unit variance [5] and initial values of the phases are generated as $\theta_i(0) \in [0, 1]$. The change of phase θ_i over time is shown in Fig. 2. It shows that the phases are diverging with time. From analysis, we obtained the equilibrium point and Jacobian at equilibrium point revealed one eigenvalue with positive real part which is destabilizing the system. For stable operation, it is very important to have synchronization in the network. Hence, a control $f_i(t) = F_i \sin(\phi_i - \theta_i)$ [5] is added, where F_i is the control gain of i^{th} node and ϕ_i is the target phase.

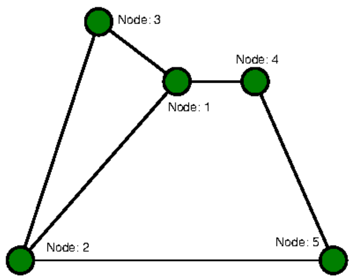

Figure 1: Network structure with five nodes

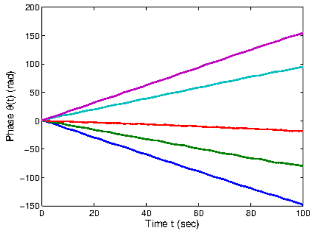

Figure 2: Phase variation with time in five node Scale free network.

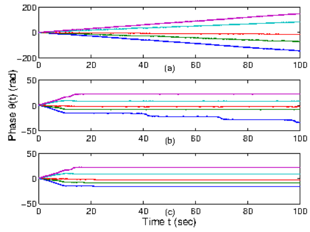

Figure 3: Phase variation with time in five node Scale Free Network with control. (a) with $\alpha = 1$, (b) with $\alpha = 3.2$ and (c) with $\alpha = 3.3$

The minimum control value can be given by $F_{i,min}$ [5],

$$F_{i,min} = K\sum_{j=1}^{N} A_{ij}[|\cos(\theta_j^* - \theta_i^*)| - \cos(\theta_j^* - \theta_i^*)] \tag{2}$$

After applying the control the new dynamics is,

$$\dot{\theta}_i = \omega_i + K\sum_{j=1}^{N} A_{ij} sin(\theta_j - \theta_i) + F_i \sin(\theta_i^* - \theta_i) \tag{3}$$

To simulate the system in eqn. 3, we considered the same K, ω_i and $\theta_i(0)$, which are used to simulate eqn. 1. We also set ϕ_i to θ_i^* which is the equilibrium point of the system in eqn. 1. The control is turned on at $t = 10\ sec$ and the change of phase over time is shown in Fig. 3. In our analysis, we used $F_{i,min}^* = \alpha F_{i,min}$, where α is the scale factor and $\alpha \geq 1$. When α is 1 as considered in Fig. 3(a), which relates to the minimum control [5] as referred in eqn. 2, the system's phase responses are still diverging. In spite of a control, phases are diverging for a value $\alpha = 3.2$ as shown in Fig. 3(b). In Fig. 3(c) it is observed that divergence of phases are gone with $\alpha = 3.3$.
In this analysis, we focused on controlling phase of each node of the network. We used a small value of network coupling strength which is kept constant throughout the analysis. It may happen that an improvement of the coupling strength is limited for a physical system. However, in this work we have shown that the system can still be stabilized by selecting a proper α value. In future, we will work on to find a minimum α which will stabilize the network. Moreover, we envisage to obtain final state of phase of each node for the following two cases by, (i) setting a desired phase of the whole network, (ii) setting a desired phase of each node.

References

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