Dynamics of slow and fast systems on complex networks

Kajari Gupta and G. Ambika *

We study the interplay between time scales of dynamical systems connected as a complex network, where out of N systems m evolve on a slower time scale. We take the case of 1) random network (where the probability of having a connection between ith and jth node is p, where, $0 \le p \le 1$) and 2) an all to all network (where all nodes are connected to all other nodes). The subset of oscillators of lower timescale in the network is defined as S. The equations of N dimensional systems are given by

$$\dot{X}_i = \tau_i F(X_i) + G\epsilon\tau_i \sum_{j=1}^{N} A_{ij}(X_j - X_i) \tag{1}$$

where $\tau_i = \tau$ if $i \in$ S, $\tau_i = 1$ otherwise. G is an N x N matrix which decides which variables are to be coupled. Here we take G = diag(1, 0, 0 ..) which means x variable of the i^{th} oscillator is coupled diffusively with the x variable of j^{th} oscillator. A_{ij} is adjacency matrix of the network. As a main result we discuss the phenomena of amplitude death and various transitions to it achieved by the nodal dynamics.

We consider a random network following the equation given in eqn.1 where $A_{ij} = 1$ with a probability p. In this case we take several realizations of network to calculate how many of the realizations go to a full amplitude death state. Calling it the fraction of realization f, we observe the transition of f with p, for different values of m. We observe that there is an optimum value of m where amplitude death occurs at the lowest possible p. This is shown in the Fig. 1. We also study these properties by taking three types of probability of connections within the network. p_1 denoting the connectivity between slow to slow systems, p_2 denoting connectivity within slow to fast systems and p_3 denoting connectivity within fast to fast systems. We observe there can be amplitude death state for the bipartite network, i.e. p_2 is non zero and $p_1 = p_3 = 0$. However having a positive value of p_1 and p_3 helps the whole network to go to amplitude death state at an earlier value of p_2.

We also take the special case of all to all network where $A_{ij} = 1$ for $i \neq j$. When the number of slow systems m is very low the systems go into different dynamics such as separating into clusters of slow and fast systems, state of two frequency etc., depending upon the time scale mismatch and coupling strength. As m increases after a threshold all the systems can go to a state of amplitude death. We isolate the region for amplitude death shown in Fig. 2. We observe that there exists a range of m/N ratio for amplitude death to occur in the network. The transitions to amplitude death is quantified and found to be similar to the work done earlier for two coupled slow and fast systems[1].

*Indian Institute of Science Education and Research, Pune 411008, India

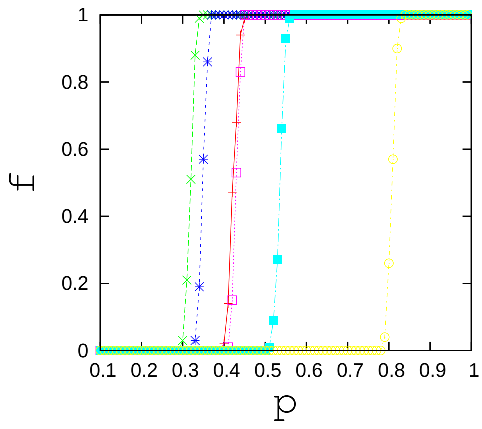

Figure 1: Transition curve for fraction of realisation f with varying probability (p) of connection in the random network, from no amplitude death state to full amplitude death state for different number of slow systems, m=30(red), 40(green), 50(blue), 60(pink), 70(sky blue), 80(yellow).

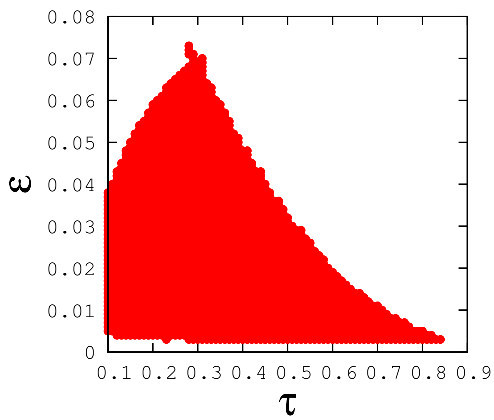

Figure 2: Region of amplitude death in τ, ϵ plane for m=50 and N=100 in an all to all network.

References

[1] Gupta, K. and Ambika, G. Eur. Phys. J. B, 89: 147 (2016)