Interesting multistabilities in coupled oscillators with attractive and repulsive coupling

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Coupled systems exhibit different interesting collective dynamics such as synchronization, clustering, oscillation quenching and so on and they have been extensively studied in various fields of science and technology [1]. In biological and chemical networks, different forms of coupling are important tools in controling/regulating the dynamics. In this aspect, the combinations of attractive and repulsive coupling play crucial roles in determining important evolutions in natural systems [2],[3]. In this presentation, we present the dynamics observed in the coupled system of attractive and repulsive coupling.

As detailed studies on two coupled systems may help to understand the origin of different collective patterns observed in the network case, we consider the attractive and repulsive coupling between two identical limit cycle oscillators. Let ϵ_1 be the strength of the attractive coupling and ϵ_2 be the strength of the repulsive coupling. In the absence of repulsive coupling, the system shows in-phase synchronized (IPS) oscillations for lower values of ϵ_1 and a pair of inhomogeneous steady states are found to be stabilized in addition to the IPS oscillations, for higher values of ϵ_1. In the absence of attractive coupling, the system is found to exhibit out-of-phase (OPS) oscillations for all values of ϵ_2.

Now in the presence of both attractive and repulsive coupling, we delineate the stable regions of different dynamical states in Fig. 1. We first consider the stabilities of IPS and OPS oscillations with respect to the combination of ϵ_1 and ϵ_2. By fixing ϵ_2 to smaller values, the increase in ϵ_1 induces destabilization of OPS oscillations and immediate stabilization of IPS oscillations. By keeping ϵ_2 to still more higher values, we observe that the variation in ϵ_1 induces a multistability between OPS and IPS states. The arise of such multistable regions can be understood interms of the competition between the attractive and repulsive coupling. For further higher values of ϵ_2, we observe that the IPS state is not stable for any value of attractive coupling strength. This shows the inability of attractive coupling in stabilizing IPS oscillations in the presence of strong repulsive coupling. Similarly, if one fixes ϵ_1 and varies the value of ϵ_2, one observes direct transition from IPS state to OPS state for lower values of ϵ_1. But by increasing ϵ_1, interestingly the OPS regions are widened

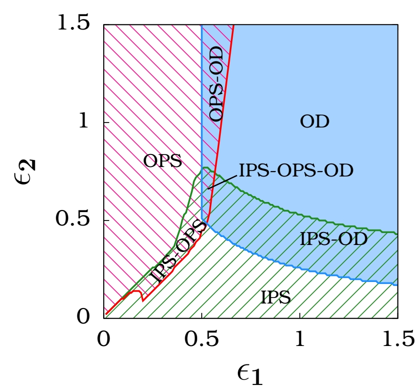

Figure 1: The stable regions of different dynamical states in ϵ_1-ϵ_2 space.

and it gives rise to multistability. Such widening of OPS region by the attractive coupling is interesting as the latter often suppresses the OPS region.

Considering now the stability of oscillation death (OD) states, Fig. 1 shows that the OD regions are widened even by the repulsive coupling. Note that in the absence of ϵ_1, the increase in ϵ_2 never stabilizes the OD state. But in the presence of ϵ_1, the variation in ϵ_2 widens the OD region. Also we note that the widening of OD regions with respect to ϵ_2 is limited, as the increase in ϵ_2 beyond a critical value do not alter the OD range as seen in Fig. 1. One can also find that there exists interesting multistabilities between IPS and OD states and between OPS and OD states. Due to the above multistabilities, we observe the interesting coexistence of all the three states (that is, IPS, OPS and OD states) at the junction of the stable region of these three states. The obtained results may provide insight on the origin of different collective dynamical states observed in the large networks.

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