Chaotic behaviour of the three-state variable rate and state friction model

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In this talk, we present chaotic behavior of the three state variables rate and state friction (3sRSF) model with spring-mass sliding system. This nonlinear behaviour of the rate and state friction (RSF) model has been mainly done with the two state variables rate and state friction (2sRSF) considering the slip law. It is to be noted that chaotic effect is not seen with the aging law of the RSF model. The governing system of differential equation for the 3sRSF is derived in the non-dimensional terms as following

$$\begin{aligned} \frac{d\phi}{dT} &= e^{\phi}[(1-\rho_1)\hat{\theta}_1 + (\rho-\rho_1)\hat{\theta}_2 .. \\ &+ (\beta_1 + \rho\beta_2 + \rho_1\beta_3 - \rho)\phi + \rho_1 f - k] + ke^{\phi_0} \\ \frac{df}{dT} &= k\left(e^{\phi_0} - e^{\phi}\right) \\ \frac{d\hat{\theta}_1}{dT} &= -e^{\phi}\left(\hat{\theta}_1 + \beta_1\phi\right) \\ \frac{d\hat{\theta}_2}{dT} &= -\rho e^{\phi}\left(\hat{\theta}_2 + \beta_2\phi\right) \end{aligned} \qquad (1)$$

Linear stability analysis of Eq.(1) shows that critical stiffness, at which dynamical behaviour changes from stick-slip motion to steady sliding or vice-versa, increases with number of state variables. Further effect of friction parameters such as β_1, β_2, β_3, ρ and ρ_1 and in Eq.(1) on its critical stiffness increases linearly. We have also simulated numerically Eq.(1) with MATLAB using ode23s to study the non-linear or chaotic behaviour of the 3sRSF model. Nonlinear tools such as phase trajectory, Poincare section, bifurcation map and Lyapunov exponents were used in the study. Fig.1 present the bifurcation map obtained by solving Eq.(1) . An interesting observation in the map that frictional stress and corresponding slip velocity at the slip interface changes from periodic oscillations to chaotic oscillations as stiffness of the sliding system decreases. Consequently, this results in a sudden increase of stress amplitude thus the nucleation of earthquake process occurs. This observation is qualitatively similar to the chaotic nature of the sliding mass with the 2sRSF in which magnitude of frictional stress as well as slip velocity increases with decrease in stiffness [1]. Fig.1

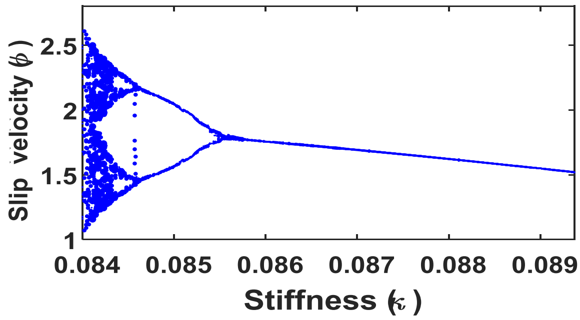

Figure 1: 1 Bifurcation diagram for spring stiffness k = 0.87 to 0.084, β_1 = 1.0,β_2 = 0.84,β_3 = 0.38, ρ = 0.048 and ρ_1 = 0.034 for initial condition [0,0,0,0].

presents the evidence that both nucleation and propagation of an earthquake are basically unpredictable and irregular phenomena [1]. We have also compared the linear and non-linear behavior between the 2sRSF and the 3sRSF models. For instance, critical stiffness increases with number of state variables in the 2sRSF as well. Moreover, the route of chaos is the same for both 2sRSF and 3sRSF models, that is, period doubling. But periods eight and sixteen motion are not observed in the present system which are unlike to the 2sRSF model [2].More significantly, Lyapunov exponents (LEs) of the 2sRSF are reported to be one positive, one negative and one zero. The 3sRSF model, in contrast, shows all four LEs are positive including one LE is very near to zero. Thus the 3sRSF is more chaotic than the 2sRSF.

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