An experimental study on control of bifurcation-delay in slow passage effect

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Bifurcation in a dynamical system is characterized by its qualitative response to a system parameter. When a system parameter varies with time continuously, bifurcation is delayed as a result of inertia in the response of the system. This phenomenon is called slow passage effect or delayed bifurcation [1]-[3]. The consequence of the slow passage effect is that bifurcation does not occur at the true bifurcation point and a delay is observed in transitions from one dynamical state to another dynamical state. The slow passage problem occurs through the several bifurcations including Hopf bifurcation, pitchfork bifurcation, saddle center bifurcations and through the resonances. Very recently, the same authors reported the experimental observation of slow passage through Hopf and pitchfork bifurcations in a parametrically driven Murali-Lakshmanan-Chua (PDMLC) oscillator [4]. The results showed that the delay in Hopf and pitchfork bifurcations increase when the rate of change of control parameter decreases and also showed that the delay obeys a power law relation with external frequency.

On the other hand, bifurcation control has intrigued many researcher from diverse fields. In order to achieve some desirable dynamical behaviors one has to modify the bifurcation properties of a given nonlinear system. This can be achieved by designing a controller through bifurcation control. In particular the delayed feedback control is widely used in mechanical and in an electronic facilities. At the same time minimize the bifurcation delay in transition from one state to another state is an interesting task and it is also an important problem in control applications.

Usually, the presence of delay enhance the dimension of systems dynamics and, thus, the complexity of the system. Many studies show that the time delay in various dynamical systems causes not only a qualitative but also a quantitative effect on its dynamics. Therefore the investigation about the delay causes a various dynamical behavior in a system becomes more important. In this connection, we discuss the impact of time delay as well as the influence the noise on slow passage problems and how to eliminate the bifurcation delay in Hopf and pitchfork bifurcations.

In this talk, specifically we discuss the phenomenon of slow passage effect which occur through the Hopf and pitchfork bifurcations in controlled parametrically driven oscillators in the presence of delayed self-feedback. We have chosen the parametrically driven Murali-Lakshmanan-Chua (PDMLC) oscillator as the prototype [5] and discuss the stability and bifurcations as a function of feedback delay. We identify that the delay in Hopf bifurcation depends on the strength of the time delay coupling as well as the magnitude of the delay. Interestingly, we find that *feedback delay* effectively controls the *bifurcation delay* of the Hopf and pitchfork bifurcation: Under certain condition, which results in the reduction of bifurcation delay. We carry out a linear stability analysis to predict how feedback delay affects the stable zone.

By considering the case of Hopf bifurcation, bifurcation delay is increased in falling edge of the amplitude of the external force for an increase of self-feedback delay. Whereas in the case of raising edge, bifurcation delay is decreased for an increase of self-feedback delay. In the case of pitchfork bifurcation, the delay in bifurcation decreases as the time delayed feedback increases in both falling and raising edges of the amplitude of the external force.

Further, in a real system noise is ubiquitous and omnipresent which drastically change (or) enhance the response of the system in their dynamics under appropriate condition. Through a influence of additive white noise, we can also eliminate the bifurcation delay in both Hopf and pitchfork bifurcations. Such investigation on noise dynamics is significant to understand the processes which occurs in nature. These results are illustrated numerically and corroborated with real time electronic circuit experiments.

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