Controlling the Basin of Attraction of Period-1 Rotation of a Horizontally Excited Parametric Pendulum

S. Das and P. Wahi *

The equation of motion of a horizontally excited parametric pendulum is

$$\ddot{\theta} + c\dot{\theta} + a\sin\theta - b\cos t\cos\theta = 0\,, \quad (1)$$

which has no fixed points and has period-1 solutions as the simplest solution. Depending on the choice of parameters a, b and c, Eq. (1) can have two qualitatively different periodic solutions viz. oscillation and rotation. There is a growing interest in rotating solutions as they can be used for energy harvesting. For small values of b, only oscillatory solutions exist and rotation appears through a saddle-node bifurcation with increasing b. In general, period-1 rotation always coexists with some oscillatory solution and the basin of attraction of rotation does not extend to the entire initial condition space. For example, at $a = 0.5, b = 0.1$ and $c = 0.03$, a stable period-1 oscillation coexists with period-1 rotation reducing its basin of attraction (see fig. 1).

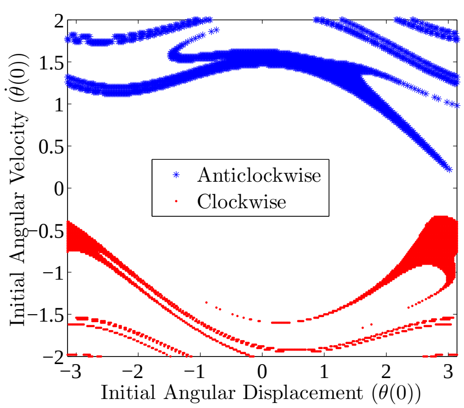

Figure 1: Basins of attraction of rotations. Set of parameters: $a = 0.5, b = 0.1$ and $c = 0.03$.

Our goal is to ensure robust initiation of period-1 rotation from all initial conditions. We employ a delayed control, an extension of Pyragas' control [1] to period-1 rotation, as

$$\ddot{\theta}+c\dot{\theta}+a\sin\theta-b\cos t\cos\theta = K[\theta(t-2\pi)-\theta(t)+2\pi\Lambda], \quad (2)$$

where $\Lambda = \mathit{Round}\left[\dfrac{\theta(t)-\theta(t-2\pi)}{2\pi}\right]$ (rounded to nearest integer) and K is the control gain. For stability studies, we linearize about $\Theta = \{$period-1 oscillation and rotation$\}$ to get

$$\ddot{\eta}+c\dot{\eta}+(a\cos\Theta+b\cos t\sin\Theta)\eta = K[\eta(t-2\pi)-\eta(t)], \quad (3)$$

The dominant Floquet multipliers as function of control gain (K) corresponding to period-1 attractors are calculated by applying semi-discretization method [2] to Eq. (3). To get an analytical expression for the periodic solutions (required to calculate the Floquet multipliers), we use the harmonic balance method [3]. From the variation of the dominant Floquet multiplier with K (see fig. 2), we observe that for $K \in [0.0220, 0.0279]$, period-1 oscillation is unstable but period-1 rotation remains stable. However, the basin of attraction of rotation for any value of K in this range does not fill the entire initial condition space since we get a stable quasi-periodic oscillatory solution. We perform a bifurcation study of the various oscillatory attractors to identify the control gain K at which all oscillatory attractors disappear. This information is then used to design a a delayed control with switching of the control gain between two values (one to destabilize all oscillatory attractors) while the other to stabilize the period-1 rotation. Details of these results will be presented at the conference.

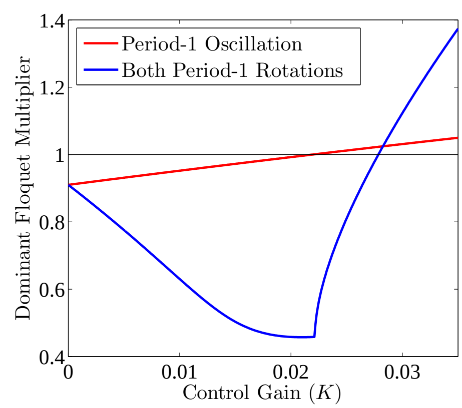

Figure 2: Dominant Floquet multiplier. Set of parameters: $a = 0.5, b = 0.1$ and $c = 0.03$.

References

[1] K. Pyragas. Continuous control of chaos by self-controlling feedback. *Physics Letters A*, 170: 421–428, 1992.

[2] T. Insperger and G. Stépán. Semi-discretization method for delayed systems. *International Journal for Numerical Methods in Engineering*, 55(5): 503–518, 2002.

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*S. Das and P. Wahi both are with Mechanical Engineering Department, Indian Institute of Technology, Kanpur-208016, email: santanu@iitk.ac.in, wahi@iitk.ac.in.