Targeting periodic solutions in chaotic systems

Vaibhav Varshney , Pooja Rani sharma, Manish Dev Shrimali, Bibhu Biswal and Awadhesh Prasad *†‡

The phenomenon of stabilization of desired periodic state around a targeted point in chaotic systems through a new variant of linear augmentation method is studied. This proposed scheme is used not only for the stabilization of a periodic orbit around a center of rotation of our choice but also with desired frequency in chaotic systems. The dynamical properties of the emergent states of the system is verified by Lyapunov exponents. It has been observed that the transition route from chaotic state to periodic state is through a boundary crisis. Mutistability is also observed in these system at the transition point.

In order to illustrate our method we first consider the chaotic Lorenz system with a new variant of linear augmentation. The dynamical equations for our system can be written as,

$$\begin{aligned}\dot{x} &= \sigma(y-x) + \epsilon u,\\ \dot{y} &= rx - y - xz + \epsilon v,\\ \dot{z} &= xy - \beta z,\\ \dot{u} &= -k_1 u - \epsilon(x-b_1),\\ \dot{v} &= -k_2 v - \epsilon(y-b_2),\end{aligned} \tag{1}$$

where, (x, y, z) and (u, v) are the state variables of Lorenz system and environment respectively. The values of the parameters of the Lorenz system are $\sigma = 10, r = 28$, & $\beta = 8/3$, at which its dynamics is chaotic. The dynamics of the environment is assumed to decay exponentially with time and the decay constant for state variable u and v are k_1 and k_2 respectively. ϵ is the strength of coupling between the system and the environment. b_1 and b_2 are the coupling parameters of the environment. Without any loss of generality we have taken $k_1 = k_2 = 0.01$.

To achieve the desired centre of rotation and frequency, we modify the coupling parameters b_1 and b_2 of the environment such that the dynamics of the environment is given by,

$$\begin{aligned}\dot{u} &= -k_1 u - \epsilon(x-\gamma_1-\alpha\cos\omega t),\\ \dot{v} &= -k_2 v - \epsilon(y-\gamma_2-\alpha\cos\omega t),\end{aligned} \tag{2}$$

where, γ_1 and γ_2 are the centre around which we want oscillation. Here, ω is the desired period of oscillation of the periodic state and α is some parameter. In this work, we have considered, $\gamma_1 = \gamma_2$ and $\gamma = \sqrt{(\gamma_1{}^2 + \gamma_2^2)}$.

*Vaibhav Varshney and Awadhesh prasad is with the Department of Physics and Astrophysics, University of Delhi, Delhi-110007, email: vaibhav.varshney1991@gmail.com, awadhesh.prasad@gmail.com

†Pooja Sharma and Manish Dev Shrimali is with the Department of Physics, Central University of Rajasthan email:poojasharma.dynamic@gmail.com, m.shrimali@gmail.com.

‡Bibhu Biswal is with Cluster Innovation centre, University of Delhi, Delhi-110007 email:bbiswal@svc.ac.in

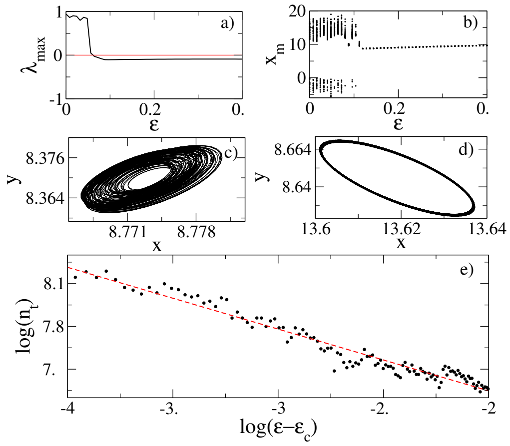

Figure 1: (a) The largest Lyapunov exponent, λ_{max}, (b) the bifurcation diagram, x_m, as a function of coupling strength ϵ at $\gamma = 15\sqrt{2}$. The projection of trajectories in x-y plane of (c) the chaotic attractor for $\epsilon = 0.008$ and (d) the periodic atractor for $\epsilon = 0.6$. (e) The variation of average transient time $\log(<n_t>)$ vs $\log(\epsilon-\epsilon_c)$ (shown in dots) for $\omega = 10$ and $\alpha = 10$. The solid line in (e) represents the linear fit to the data.

The results are shown in the Fig. 1. In Fig. 1 (a) and (b) the variation of largest Lyapunov exponent and bifurcation diagram against coupling strength ϵ is shown. The representative Chaotic and periodic attractors are shown in Fig. 1 (c) and (d). In this case, the transition is sudden and chaotic attractor disappears as the coupling strength is increased. This also give rise to sudden appearance of periodic attractor. In Fig. 1 (e), the average transient lifetime $<n_t>$ over 100 initial conditions as a function of $(\epsilon-\epsilon_c)$ is plotted. Here, ϵ_c the critical value of parameter ϵ where the transition from chaotic state to periodic state occur in the system. In this case, $\epsilon_c = 0.14$. Clearly, from the Fig. 1(e), $<n_t>$ shows a power law behavior with $(\epsilon-\epsilon_c)$. This can be approximated as $<n_t>\sim(\epsilon-\epsilon_c)^{-\beta}$, where β is the critical exponent. Fitting the numerical data we found that the value of the exponent is $\beta = 0.48$.

Full paper submitted in journal

References

[1] P. R. Sharma, A. Sharma, M. D. Shrimali and A. Prasad, *Physical Review E* **83** 067201 (2011).

[2] K. Kiyono and N. Fuchikami, *J. Phy. Soc. Japan* **71** 49 (2002).

[3] G. Katriel, *Physica D* **237** 2933 (2008).

[4] V. Varshney, P. R. Sharma, M. D. Shrimali, B. Biswal and A. Prasad *(under preparation)*