Entanglement transitions in eigenstates of interacting chaotic systems

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Entanglement in the eigenstates of quantum non-integrable systems can be nearly as large as the maximum allowed, as these are statistically described by random states in product spaces. That subsystems of isolated systems can behave as if in thermal equilibrium due to entanglement has been proposed as an alternative to the foundations of statistical physics. One context in which entanglement in eigenstates increases from zero to nearly maximum is when two erstwhile non-interacting completely chaotic systems start to interact.

Let H_0 be a bipartite, separable Hamiltonian, say $H_0^{(1)} + H_0^{(2)}$ but each of which is nonintegrable and if there are classical limits, these correspond to fully chaotic systems. For instance these could be two particles in a quantum dot shaped like the Bunimovich stadium. The eigenvalue fluctuation statistics before interaction is Poissonian, characterized by small nearest neighbor spacings, although the subsystems are chaotic. On adding a small interaction, eigenstates start getting entangled, and the eigenvalue fluctuations start a transition to a global random matrix theory (RMT) statistics characterized by level repulsion. This transition in the eigenvalue fluctuation was recently shown [1] to be a rapid and universal one that depended on only one parameter, which was simply calculable from the interaction.

The eigenstates of H_0 are complex but unentangled. However even small interactions are sufficient to create nearly maximally entangled states, and the purpose of this work is to characterize the universal way in which this transition happens as a function of strength of the interaction. An exponential and universal increase in the entanglement is found which asymptotically approaches the RMT value relevant to the whole system. The universal increase is governed by (square-root of) the same scaling parameter (Λ) that governs the transition in the nearest neighbor spacings. Perturbation theory related arguments are advanced that give a simple but surprisingly accurate analytical description of the entire transition.

Let the reduced density matrix of the first subsystem for the eigenstate labeled j be $\rho_j = \mathrm{tr}_2(|\phi_j\rangle\langle\phi_j|)$. As the complete state is pure, its entanglement is characterized by the eigenvalues of ρ_j. In particular the von Neumann entropy $S_1 = -\mathrm{t}r_1(\rho_j \ln \rho_j)$ is considered a unique measure, as it quantifies the entanglement that can be concentrated by local operations. The quantities

$$S_k = \frac{1-P_k}{k-1}, \quad P_k = \mathrm{tr}_1(\rho_j^k) \tag{1}$$

simply related to the k-th order moments P_k, are the so-called Tsallis entropies. Of them the linear entropy corresponding to $k = 2$ is often used as a simpler measure of entanglement than the von-Neumann which corresponds to the limit of $k \to 1$. The state $|\phi_j\rangle$ is unentangled iff the state ρ_j is pure, when all the S_k vanish.

It is shown in this paper that the transition when Λ increases from the uncoupled limit of 0 is captured by the entropies in a remarkably simple form

$$\langle S_k(\Lambda)\rangle = \left(1 - \exp\left(-\frac{\alpha(k)}{\langle S_k^\infty\rangle}\sqrt{\Lambda}\right)\right)\langle S_k^\infty\rangle, \tag{2}$$

where

$$\alpha(k) = \pi\frac{\Gamma(k-\frac{1}{2})}{\Gamma(k)}, \quad \langle S_k^\infty\rangle = \frac{1 - C_k N^{1-k}}{k-1}, \tag{3}$$

and C_k are Catalan numbers. The $\langle\cdot\rangle$ represent a ensemble or spectral averge and $P_k^\infty = C_k N^{1-k}$ are moments of the Marcenko-Pastur distribution that determines the large N density of the eigenvalue of ρ_j in the fully mixed RMT limit. The asymptotic entropies $\langle S_k^\infty\rangle$ are reached at the end of the transition and while the form above is valid for $k > 1$, $\langle S_1^\infty\rangle = \ln N - \frac{1}{2}$.

Two models, both being unitary operators of the form $U = (U_1 \otimes U_2)\, U_{int}$ relevant to coupled Floquet systems were considered: (1) a RMT model when $U_{1,2}$ are random unitary matrices of dimension N and the interaction U_{int} is a diagonal matrix with random phases is considered and (2) coupled kicked rotors as a dynamical system model in the regime where the uncoupled rotors were very chaotic. The agreement between these models and the entropy formulas derived above are surprisingly good. The entanglement starts from zero and for $\Lambda \sim 2$ the linear entropy and higher order ones are close to their asymptotic value. The slowest corresponds to the von Neumann entropy that is a more conservative measure of information, and here the Λ at which the transition is complete increases as $(\log N)^2$, unlike the entropies for $k > 1$.

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References

[1] Shashi C. L. Srivastava, Steven Tomsovic, Arul Lakshminarayan, Roland Ketzmerick, and Arnd Bäcker, *Universal scaling of spectral fluctuation transitions for interacting chaotic systems*, Phys. Rev. Lett. **116**, 054101 (2016).

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