Curl Force Dynamics

M V Berry and Pragya Shukla
H H Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, UK,
Department of Physics, Indian Institute of Technology, Kharagpur, India,
email: shukla@phy.iitkgp.ernet.in

Curl forces, governing the acceleration of particles in Newtonian dynamics, are position-dependent but not derivable from a potential.A physical example is forces exerted on small particles by light. Although non conservative, curl forces are not dissipative because volume in the position-velocity state space is preserved. When the force has rotational symmetry, for example when generated by a single optical vortex, particles spiral outwards and escape, even with an attractive gradiant force, however strong. Without rotational symmetry, and for dynamics in the plane, possible constants of motion can be sought numerically using the Volume of Section (VoS): dots representing times along an orbit satisfying a condition (e. g. crossing the x axis), in the three-dimensional space of the remaining variables. For some curl forces, e.g., optical fields with two opposite-strength vortices, dots in the VoS lie on surfaces, indicating a hidden constant of motion. For other curl forces, e.g., those from four vortices, the dots form ‘dust cloud’ patterns, apparently exploring volumes in an un familiar kind of chaos, suggesting that no constant of motion exists. The dynamics generated by opticalvortices could be studied experimentally.