Single-User MIMO System, Painleve Transcendants and Double scaling

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In this paper we study a particular Painleve V (denote as P_V) that arises from Multi-Input-Multi-Output (MIMO) wireless communication systems. Such a P_V shows up, because of its intimate relation with the Hankel determinant that describes the moment generating function (MGF) of the Shannon capacity. This originates through the multiplication of the Laguerre weight or the Gamma density, $x^\alpha e^{-x},\ x > 0,\ \ \alpha > -1$ by $(1 + x/t)^\lambda,\ \ t > 0$. Here the λ parameter which "generates" the Shannon capacity. See Yang Chen and Matthew McKay, IEEE Trans. IT, vol.(58) (2012) 4594–4634. It was found that the MGF has an integral representation as a functional of $y(t)$ and $y'(t)$, where $y(t)$ satisfies the "classical form" of P_V. In this paper, we consider the situation where n, the number of transmit antenna, (or the size of the random matrix), tends to infinity, and the signal-to-noise ratio (SNR) P tends to infinity, such that $s = \frac{4n^2}{P}$ is finite. Under such double scaling the MGF, effectively an infinite determinant, has an integral representation in terms of a "lesser" P_{III}. We also consider the situation where $\alpha = k + 1/2,\ \ k \in \mathbb{N}$, and $\alpha \in \{0, 1, 2, ...\}\ \ \lambda \in \{1, 2, ..\}$, linking relevant quantity to the Sine-Gordon equation and certain discrete Painleve-II.
From the large n asymptotic of the orthogonal polynomials, that appears naturally, we obtain the double scaled MGF for small and large s, together with the constant term in the large s expansion. With the aid of these, we derive a number of cumulants and find that the capacity distribution function is non-Gaussian.