Fractional soliton solutions in a purely nonlinear complex Ginzburg-Landau equation through a nonlinear discrete transmission line

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Solitons are stationary pulses which propagate in nonlinear dispersive media. They keep their stable wave forms due to a dynamical balance between the nonlinear and the dispersive effects. There are only a few systems where solitons are easily and directly observed in controlled laboratory experiments. Nonlinear electrical transmission lines (NLTLs) are good examples of such systems and are very convenient tools for the study of wave propagation in nonlinear dispersive media. A nonlinear transmission line is comprised of a transmission line periodically loaded with varactors, where the capacitance nonlinearity arises from the variable depletion layer width, which depends both on the dc bias voltage and on the ac voltage of the propagating wave. For example, the model shown in Fig. 1 is a lossless discrete nonlinear transmission line made of ladder-type LC circuits containing constant inductors and voltage-dependent capacitors. Each cell contains a linear inductance L_1 in parallel with a linear capacitance C_S in the series branch and a linear inductance L_2 in parallel with a nonlinear capacitance $C(V)$ in the shunt branches. The nonlinear capacitors are usually reverse-biased capacitance diodes. In typical experiments the number of identical sections is between 50 and 1000. Using reductive perturbation method and complex expansion for the nonlinear transmission line shown on Fig. 1, one can easily derives the following version of the complex Ginzburg-Landau equation [1]:

$$i\partial_t\Psi = \alpha\partial_{xx}\Psi + \beta\Psi + \gamma_1|\Psi|^4\Psi + i\gamma_2|\Psi|^2\partial_x\Psi + i\gamma_3\Psi^2\partial_x\Psi^*, \qquad (1)$$

where the coefficients α, β, γ_1, γ_2, and γ_3 are real and are given in terms of the transmission line's parameters. Equation (1) is called the derivative nonlinear Schrödinger equation with potential or the cubic-quintic Ginzburg-Landau equation. The derivative nonlinear Schrödinger equation (1) is used for modeling of wave processes in different physical systems such as nonlinear optics, circular polarized Alfvéen waves in plasma, Stokes waves in fluids of finite depth, etc. In nonlinear optics, Eq. (1) can be derived in a systematic way by means of the reductive perturbation scheme as a model for single mode propagation. In the context of waveguides as optical fibers, t usually corresponds to the propagation distance of the electric field envelope Ψ of an optical beam along the fiber, x plays the role of the time, the term $\gamma_1|\Psi|^4\Psi$ models the nonlinear Kerr effect, while $\gamma_2|\Psi|^2\Psi_x$ and $\gamma_3\Psi^2\Psi_x^*$ are the nonlinear dispersion contributions.

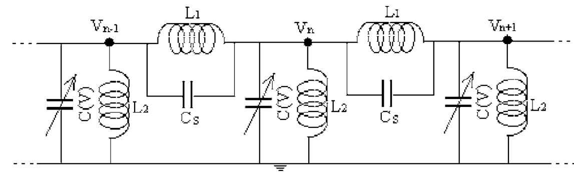

Figure 1: Schematic representation of the NLTL.

By using traveling wave coordinates and the first integral method, we can transformed Eq.(1) into an elliptic ordinary differential equation of the following form

$$(\rho^{'})^2 = -\frac{4K_1^2}{\alpha^2} + K_2\rho + E\rho^2 + Fp^3 + G\rho^4, \qquad (2)$$

where $E = \frac{2K_1(\gamma_2-\gamma_3)-v^2-4\alpha(\beta+lv-q)}{\alpha^2}$, $F = \frac{v(\gamma_3-\gamma_2)}{\alpha^2}$ and $G = \frac{(\gamma_2+\gamma_3)(5\gamma_3-3\gamma_2)-16\alpha\gamma_1}{3\alpha^2}$.

After that, we employ a fractional Möbius transformation [2] to Eq.(2) and systematically obtain both bright and dark localized pulses, as also non-singular solutions. We consider the following fractional transformation (FT)

$$\rho(\zeta) = \frac{A + Bf(\zeta,m)^{\mu}}{1 + Cf(\zeta,m)^{\mu}}, \qquad (3)$$

where A, B and C are real constants, μ is an integer, and $f(\zeta,m)$ is a Jacobian elliptic function with the modulus parameter $m(0 \leq m \leq 1)$. Here we describe two families of localized solutions of Eq.(2) for $\mu = 2$, $f(\zeta,m) = cn(\zeta,m)$ and $f(\zeta,m) = sn(\zeta,m)$ in the FT. We find that for selecting some values of the coefficients of the equation and for some parameters of solutions, the profile of solutions can be well controlled.

References

[1] E. Kengne and K. Kum Cletus. Modulational instability in nonlinear bi-inductance transmission line. *Int. J. Mod. Phys. B*, 19: 3961-3983, 2005.

[2] T. S. Raju and P. K. Panigrahi. Self-similar propagation in a graded-index nonlinear-fiber amplifier with an external source. *Phys. Rev. A*, 81: 043820, 2010.

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