Solitary waves in multicomponent nonlinear Helmholtz equations

K. Tamilselvan, T. Kanna and Avinash Khare *

We study different classes of solitary waves of the multicomponent nonlinear Helmholtz (mNLH) equations describing nonparaxial ultra broad beam propagation in planar waveguides. The mNLH equations are solved using Lamé polynomials of order 1, 2 and 3 in the hyperbolic limit. The effect of nonparaxiality on the speed, pulse width and amplitude of the nonlinear waves is analysed in detail. Particularly a mechanism for tuning the speed (modulus of velocity) by altering the non-paraxial parameter is proposed.

The great deal of interest on the multicomponent solitary waves in various nonlinear media over past two decades can be attributed to their ubiquitous appearance in different physical systems as well as their multifaceted applications in the field of nonlinear optics, Bose-Einstein condensates, bio-physics, etc [1]. In general, the nonlinear optical media can support multicomponent solitary waves which are governed by the multicomponent nonlinear Schrödinger equations (mNLS). The mNLS equations stem naturally from the Maxwell's equations by employing the paraxial wave approximation or slowly varying envelope approximation (SVEA). Under this approximation the second order derivative of normalized wave with respect to propagation direction is ignored. Here a beam broader than its carrier wavelength, with moderate intensity and is propagating in (or at a negligible angle) with respect to the reference direction is said to be a paraxial beam. If the beam fails to satisfy at least any one of the aforementioned properties is said to be a nonparaxial beam.
We consider the following general dimensionless mNLH equations describing interaction of three obliquely propagating incoherently coupled optical fields in planar waveguide [2]-[3]

$$iq_{j,t} + \kappa\, q_{j,tt} + \frac{1}{2}\, q_{j,xx} + \sum_{l=1}^{3} (\sigma_l |q_l|^2) q_j = 0. \qquad (1)$$

Here $q_j, j = 1, 2, 3$ are the three components of the envelope optical fields, subscripts t and x represent the longitudinal and transverse coordinates respectively.
In this paper, we are constructing the hyperbolic solutions in terms of Lamé polynomials of order from one to three in the hyperbolic limit. To construct the hyperbolic solutions of system (1), we introduce the traveling wave ansatz as

$$q_j = f_j(u) e^{i\alpha_j}\,. \qquad (2a)$$

*K. Tamilselvan (email: tamsel.physics@gmail.com) and T. Kanna (email: kanna_phy@bhc.edu.in) are with the Post Graduate and Research Department of Physics, Bishop Heber College, Tiruchirapalli-620 017. India. Avinash Khare (email: khare@physics.unipune.ac.in) is with the INSA senior scientist, Department of Physics, Savitribai Phule Pune University, Pune – 411007, India.

where

$$u = \beta(x - vt + \delta_0), \quad \alpha_j = (k_j x - \omega_j t + \delta_j) \qquad (2b)$$

In Eq. (2) $f_j, j = 1, 2,$ are real functions of t and x while β, δ_0 and δ_j $j = 1, 2, 3$ are real constants, ω_j are frequencies of the three components of mNLH system, v is the velocity and, k_j is the wave number of the j^{th} component q_j. By applying the ansatz (2) to system (1), we find f_j satisfy Lamé equations of order 1, 2 and 3 in the hyperbolic limit. Hence we obtain three different families of solutions corresponding to these Lamé polynomials of order 1, 2 and 3 in the hyperbolic limit. The resulting first order, second order and third order solitary wave solutions in terms Lame polynomials of order 1, 2 and 3 in the hyperbolic limit are given below and these solutions are associated with their constraint conditions but which are not given in this paper [3].
First order solitary wave solutions: The first order solitary waves of the mNLH system (1) consist of four distinct solutions obtained by different combination of simple bright ($\text{sech}(u)$) and dark ($\tanh(u)$) solitary waves. Each of this four class of solitary waves have fifteen parameters with six conditions, so it admits nine arbitrary parameters. In what follows, we have tabulate the first order solitary waves of the mNLH system (1).

Table 1: First order hyperbolic solutions

S.No	$f_1(x,t)$ $q_1 = f_1(x,t) \times e^{i\alpha_1}$	$f_2(x,t)$ $q_2 = f_2(x,t) \times e^{i\alpha_2}$	$f_3(x,t)$ $q_3 = f_3(x,t) \times e^{i\alpha_3}$
(1)	$A\ \tanh(u)$	$B\ \tanh(u)$	$C\ \tanh(u)$
(2)	$A\ \text{sech}(u)$	$B\ \text{sech}(u)$	$C\ \text{sech}(u)$
(3)	$A\ \tanh(u)$	$B\ \text{sech}(u)$	$C\ \text{sech}(u)$
(4)	$A\ \text{sech}(u)$	$B\ \tanh(u)$	$C\ \tanh(u)$

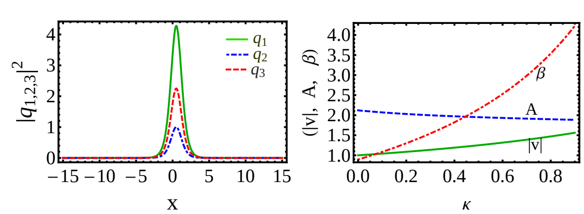

Figure 1: (a) show the intensity plot of solution (2) from Table 1, (b) show the physical parameters versus nonparaxial parameter.

For illustrative purpose, we present the intensity plot of the solution (2) and examine the role of the nonparaxial parameter on the solitary waves, we depict physical parameters (speed (modulus of velocity), pulse width ($1/\beta$) and the amplitudes first component) versus κ for the same solution in Fig. 1(b). The pulse width and amplitude A of the first component behave in an opposite manner as κ increases in Fig. 1(b). One can easily note that the speed increases as κ increases in Fig. 1(b). The amplitude of the first component is influenced by the second and third components as shown in Fig. 1(a).

Second order solitary wave solutions: Next, we shift our attention to second order solitary waves of the mNLH system. The second order solitary wave solutions of the mNLH system (1) consist of seven distinct set of solitary wave solutions resulting from the combination of the red ($\mathrm{sech}^2(u) - \frac{2}{3}$), white ($\tanh(u)\mathrm{sech}(u)$) and blue ($\mathrm{sech}^2(u)$) solitary waves. Each of these seven class of solitary wave solutions characterized by fifteen parameters with seven conditions, so it admits eight arbitrary parameters. In below, we tabulate the second order solitary wave solutions.

Table 2: Second order hyperbolic solutions

S.No	$f_1(x,t)$ $q_1 = f_1(x,t)$ $\times e^{i\alpha_1}$	$f_2(x,t)$ $q_2 = f_2(x,t)$ $\times e^{i\alpha_2}$	$f_3(x,t)$ $q_3 = f_3(x,t)$ $\times e^{i\alpha_3}$
(1)	$A\ \mathrm{sech}(u) \times \tanh(u)$	$B\ \mathrm{sech}(u) \times \tanh(u)$	$C\ \mathrm{sech}^2(u)$
(2)	$A\ \mathrm{sech}(u) \times \tanh(u)$	$B\ \mathrm{sech}^2(u)$	$C\ \mathrm{sech}^2(u)$
(3)	$A\ \mathrm{sech}(u) \times \tanh(u)$	$B\ \mathrm{sech}(u) \times \tanh(u)$	$C\ \mathrm{sech}^2(u) - \frac{2}{3}$
(4)	$A\ \mathrm{sech}^2(u) - \frac{2}{3}$	$B\ \mathrm{sech}^2(u) - \frac{2}{3}$	$C\ \mathrm{sech}^2(u)$
(5)	$A\ \mathrm{sech}^2(u) - \frac{2}{3}$	$B\ \mathrm{sech}^2(u)$	$C\ \mathrm{sech}^2(u)$
(6)	$A\ \mathrm{sech}(u) \times \tanh(u)$	$B\ \mathrm{sech}^2(u) - \frac{2}{3}$	$C\ \mathrm{sech}^2(u) - \frac{2}{3}$
(7)	$A\ \mathrm{sech}(u) \times \tanh(u)$	$B\ \mathrm{sech}^2(u) - \frac{2}{3}$	$C\ \mathrm{sech}^2(u)$

In order to illustrate the second order solitary waves, we present the intensity plot of the solution (1) given in Table 2 and its role of the nonparaxial parameter on physical parameter in Fig. 2(a) and 2(b) respectively. Here too the speed of the solitary waves increases as κ increases but the pulse width and amplitude A of the first component are decreasing as κ increases as shown in Fig. 2b. The first q_1 and second q_2 components admit double-hump bright solitary wave but third q_3 component admits standard bright solitary wave profile as shown in Fig. 2(a).

Third order solitary wave solutions: Finally, we focus our attention to third order solitary waves of the mNLH system. This third order solitary waves have four distinct class of solitary wave solutions. As in the previous order of solutions, here too each of these four set of solutions characterized by fifteen parameters with eight conditions, so it admits seven arbitrary parameters.

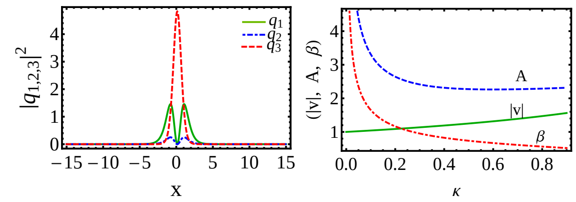

Figure 2: (a) show the intensity plot of solution (1) from Table 2, (b) show the physical parameter versus κ.

Table 3: Solutions in terms of Lamé polynomials of order 3

S.No	$f_1(x,t)$ $q_1 = f_1(x,t)$ $\times e^{i\alpha_1}$	$f_2(x,t)$ $q_2 = f_2(x,t)$ $\times e^{i\alpha_2}$	$f_3(x,t)$ $q_3 = f_3(x,t)$ $\times e^{i\alpha_3}$
(1)	$A\ \mathrm{sech}^3(u)$	$B\ \mathrm{sech}(u) \times(\tanh^2(u) - \frac{1}{5})$	$C\ \tanh(u) \times(\tanh^2(u) - \frac{3}{5})$
(2)	$A\ \mathrm{sech}^2(u) \times \tanh(u)$	$B\ \mathrm{sech}^3(u)$	$C\ \tanh(u) \times(\tanh^2(u) - \frac{3}{5})$
(3)	$A\ \mathrm{sech}^2(u) \times \tanh(u)$	$B\ \mathrm{sech}(u) \times(\tanh^2(u) - \frac{1}{5})$	$C\ \tanh(u) \times(\tanh^2(u) - \frac{3}{5})$
(4)	$A\ \mathrm{sech}^2(u) \times \tanh(u)$	$B\ \mathrm{sech}^3(u)$	$C\ \mathrm{sech}(u) \times(\tanh^2(u) - \frac{1}{5})$

In order to make clear, we have illustrated solution (1) (given in Table 3) out of four, third order solutions and plot their physical parameter versus κ in Fig. 3(a) and 3(b) respectively

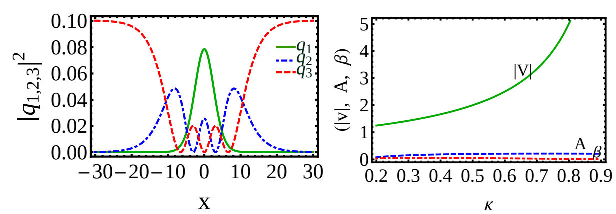

Figure 3: (a) show the intensity plot of solution (1) from Table 3, (b) show physical parameter versus κ.

Here first component q_1 admits bright solitary wave profile but second q_2 and third q_3 components admit double-hump bright and double-hump dark solitary waves respectively in Fig. 3(a). In Fig. 3(b), the speed increases as κ increase but pulse width and amplitude of the first component increase for some value of κ after that they get saturated as κ increases.

References

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