Resonant Solitons and Breathers in Higher Dimension: A Study on Long-wave–Short-wave System

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In this work, we investigate the occurrence and dynamics of resonant solitons and breathers in a higher dimensional nonlinear resonant interaction (long-wave–short-wave) system. After a brief revisit of different types of bright soliton collisions, we carry out a critical analysis to unravel the dynamics of resonant solitons/breathers resulting for infinite phase-shift. Especially, we obtain an inclined breather, time localized Akhmediev breather, space localized Ma breather and space-time localized rogue wave for different choices of soliton parameters. Also, we show the presence of resonant solitons with multiple oscillating sidebands.

Nonlinear wave interactions lead to several interesting dynamics in physical systems. Particularly, they are important in the formation of different wave structures like solitons, soliton-like structures, breathers, rogue waves, vortex solitons, etc. and they show intriguing dynamical behaviour. In higher dimensional systems, these nonlinear waves posses further interesting charecteristics [1]. In this work, we consider a higher dimensional system with nonlinear resonance interaction of long-wave and short-waves. Generally, a resonance interaction between the low-frequency long-wave (LW) and high-frequency short-waves (SWs) occurs when the phase velocity of the LW matches exactly/approximately the group velocity of the SWs [2]. One of such mathematical models is the following integrable multicomponent (2+1)-dimensional long-wave–short-wave resonance interaction (LSRI) system [3]:

$$i(S_t^{(\ell)} + \delta S_y^{(\ell)}) - S_{xx}^{(\ell)} + LS^{(\ell)} = 0, \qquad (1a)$$

$$L_t = 2\sum_{\ell=1}^{M} c_\ell |S^{(\ell)}|_x^2, \qquad \ell = 1, 2, 3, ..., M. \quad (1b)$$

In Eqn. (1), $S^{(\ell)}$ represents the ℓ-th SW, L indicates the LW and the subscripts represent the partial derivatives with respect to the evolutional coordinate t and the spatial coordinates (y and x). The above system is found to be completely integrable for arbitrary nonlinearity coefficients (c_ℓ) and arbitrary (δ) [3]. Equation (1) arises as governing equation in different contexts like nonlinear optics, water waves, Bose-Einstein condensates, plasma physics and bio-physics. The parameter c_ℓ can be related to the strength of self-phase and cross-phase modulations in nonlinear optics, while it corresponds to the inter- and intra-atomic interactions in Bose-Einstein condensates.

We have obtained the bright multi-soliton solution of LSRI system (1) and analyzed their propagation as well as collision dynamics for different choices of c_ℓ [3]. We found that the solitons exhibit both head-on and overtaking collisions in the $(x-y)$ plane, whereas in the $(x-t)$ plane they support only overtaking collisions, due to the fact that these solitons propagate with different velocities in the $(x-y)$ and $(x-t)$ planes. Importantly, the solitons in the SW component undergo two kinds of energy-sharing (inelastic or shape-changing) collision processes, namely (i) type-I energy-sharing collision for all $c_\ell > 0$ (or all $c_\ell < 0$) and (ii) type-II energy-sharing collisions for mixed-signs of c_ℓ. In such collisions, in addition to the conservation of energy in individual components, the total energy of all SW components is conserved in type-I and the difference in the energy of SW components is conserved in type-II. For all choices of c_ℓ, the LW solitons undergo only a standard elastic collisions and one can also observe elastic collisions in the SW components for special choices of soliton parameters.

Apart from the elastic/energy-sharing nature, the colliding solitons experience a phase-shift after collision. In a majority of situations, particularly in one-dimensional systems, the phase-shifts remain as an unimportant identity because of their inefficiency in showcasing novel features. But in the higher dimensional systems they can play some crucial role in the dynamics of underlying solitons [4]. Interestingly, in the present system, the phase-shifts give

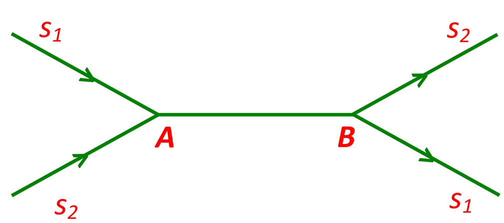

Figure 1: Schematic for the occurrence of RS/RB in the region AB in the collision of two solitons s_1 and s_2.

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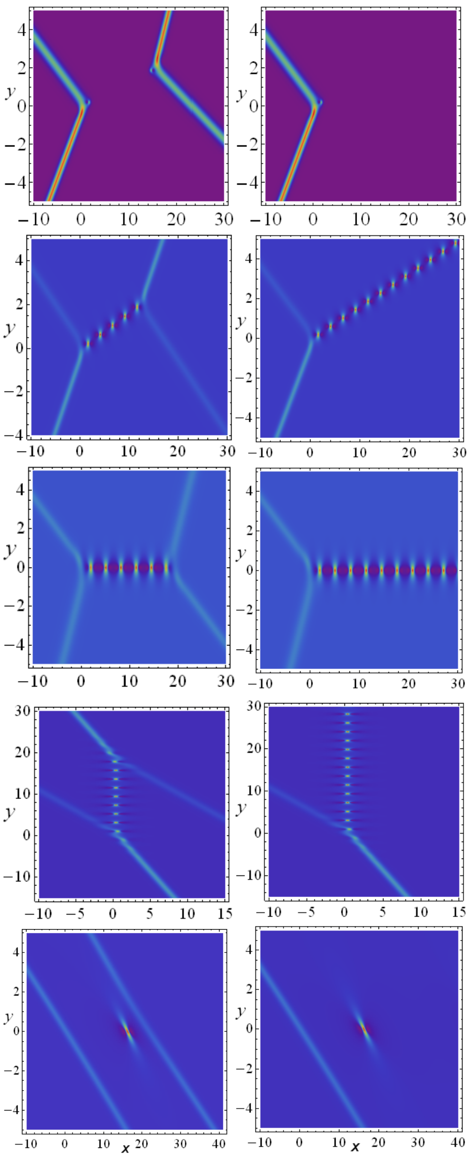

Figure 2: From top to bottom:- Zero-amplitude QRS/RS in the SW. Inclined, Akhmedieve-type, Ma-type and rogue-wave-like QRBs (left panels) and RBs (right panels) in the $(x-y)$ plane of the LW.

rise to the birth of resonant-solitons (RSs) and resonant-breathers (RBs) in the long-living interaction regime (region AB in Fig. 1) of soliton collisions. In general, these RSs/RBs occur when the phase-shift experienced by the colliding solitons becomes infinity, which display several interesting mechanism like soliton fusion, soliton fission, long-range interaction, breathers, soliton web, etc. Also, for a long and finite AB region one can observe similar patterns in that finite length.

In a two-soliton collision of the present system (1), we obtain the phase-shift Φ_j experienced by the two colliding solitons s_j, $j=1,2$, in explicit form as

$$\Phi_1 = -\Phi_2 \equiv \ln\left(\sqrt{1-\frac{\kappa_{12}\kappa_{21}}{\kappa_{11}\kappa_{22}}}\left|\frac{k_1-k_2}{k_1+k_2^*}\right|\right), \qquad (2)$$

where $\kappa_{ji} = \frac{-1}{(\omega_i^*+\omega_j)}\sum_{\ell=1}^{M}\left(c_\ell \alpha_j^{(\ell)}\alpha_i^{(\ell)*}\right)$, $j,i = 1,2$.

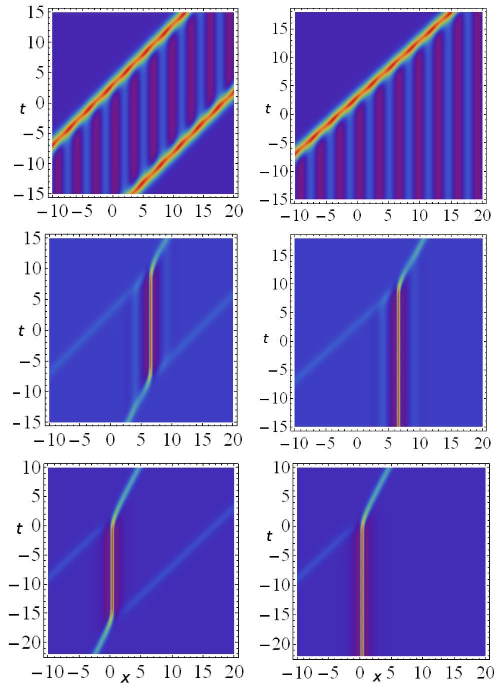

Figure 3: QRS (left panels) and RS (right panels) with side-band oscillations in the $(x-t)$ plane of th LW.

The RS can be obtained for the choice $\omega_1 = \omega_2$, which makes the phase-shifts (2) infinity, i.e., $|\Phi_j| \to \infty$. In the SW components, this RS choice produces zero-amplitude resonant state in the regime AB which lasts for infinite length. However in the LW component, we have obtained a infinite-long periodic structure in the $(x-y)$ plane and we refer to it as a resonant-breather (RB), see Fig. 2. The resonant state exhibits a stable profile with oscillating soliton side-bands in the $(x-t)$ plane, see Fig. 3. Note that the energy of SW components disappears completely and reappears in the LW component with a large amplitude periodically oscillating structure. Importantly, even if $\omega_1 \simeq \omega_2$ the length of the resonant regime AB becomes large enough to admit a finite length RS/RB without any change in their structure, which can be designated as quasi-resonant-breather (QRB)/quasi-resonant-soliton (QRS) for a better understanding. We found an interesting point that the localization of these RBs can be controlled in the 'x or in the 'y' direction by tuning the soliton parameters k_j, $j=1,2$.

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