A Randomly Perturbed DC/DC Converter

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The present paper studies the effect of small Brownian noise on a switching dynamical system modelling a first-order DC/DC buck converter [1]. The state vector of this system comprises a continuous component whose dynamics switch, based on the ON/OFF configuration of the circuit, between two ordinary differential equations (ODE), and a discrete component which keeps track of the ON/OFF configurations. Assuming that the parameters and initial conditions of the unperturbed system have been tuned to yield a stable periodic orbit, we study the stochastic dynamics of this system when the forcing input in the ON state is subject to small white noise fluctuations. For the ensuing stochastic system whose dynamics switch at random times between a small noise stochastic differential equation (SDE) and an ODE, we prove a functional law of large numbers which states that in the limit of vanishing noise, the stochastic system converges to the underlying deterministic one on time horizons which are not "too large" relative to the strength of the noise.

In more detail, the state of our system at time $t \in [0, \infty)$ is specified by a vector $z(t) \triangleq (x(t), y(t))$ taking values in $\mathscr{Z} \triangleq \mathbb{R} \times \{0, 1\}$. Here, the continuous component $x(t) \in \mathbb{R}$, which corresponds to the inductor current, is governed by one of two different ODE depending on whether the switch is ON ($y(t) = 1$) or OFF ($y(t) = 0$). Transitions from ON to OFF occur when $x(t)$ increases to a certain threshold, while OFF to ON transitions are triggered by a periodic clock signal. After tuning the parameters and initial conditions to obtain a stable periodic orbit, we study the stochastic process $Z_t^\varepsilon = (X_t^\varepsilon, Y_t^\varepsilon)$ which arises when the ODE in the ON configuration is replaced by an SDE counterpart driven by Brownian motion of size $0 < \varepsilon \ll 1$. For $z(t)$ and Z_t^ε—which can be regarded, respectively, as a point and a random variable in the metric space $D([0, \mathsf{T}_\varepsilon]; \mathscr{Z})$ of functions mapping $[0, \mathsf{T}_\varepsilon]$ to $\mathscr{Z}$ which are right-continuous with left limits [2]—we show that the distance between Z^ε and z goes to zero (as $\varepsilon \searrow 0$) in a suitable sense for time horizons T_ε of order $\mathscr{O}(1/\varepsilon^\nu)$, $0 \leq \nu < 2/3$.

Full paper Chetan D. Pahlajani, Randomly perturbed switching dynamics of a DC/DC converter, *Discrete Contin. Dyn. Syst. Ser. B*, accepted.

References

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