The generalized Lozi map: bifurcation and dynamics

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The behaviour of low-dimensional nonlinear iterative maps and flows has been extensively studied and characterized, in particular with reference to the creation of chaotic dynamics [1, 2, 3, 4]. The various scenarios or routes to chaos in such systems are by now fairly well known [1, 5, 6, 7]. The motion in higher dimensional systems—for instance the dynamics of attractors with more than one positive Lyapunov exponent and the bifurcations through which they have been created—has not been studied in as much detail even in relatively "simple" systems [4, 8, 9, 10, 11].

Our goal in the present work is to explore the transition from low– to high–dimensional dynamics in a generalized Lozi map. The Lozi map is a piecewise smooth discrete time dynamical system. Studying piecewise smooth dynamical systems is relatively simple but they present challenges unlike those of smooth counterparts. This time–delayed iterative map with a single nonlinear term, implemented by the absolute value $|.|$, also contains the original Lozi map introduced in [4] as special case. The new map has interesting feature of displaying positive Lyapunov exponents beginning from one to the dimension of the map. The transition to Furthermore we characterize the different bifurcations that occur as parameters are varied. The effects of smooth approximation [12] to piecewise smooth nature of the map is also investigated vis-a-vis bifurcations.

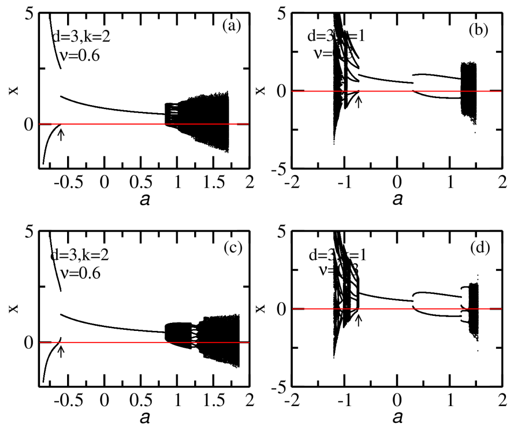

Figure 1: The bifurcation diagram for $\epsilon = 0$ (a)–(b) and $\epsilon = 0.1$ (c)–(d). Notice that bifurcations are no longer only due to border collisions: There are two borders ,one each at $\pm\epsilon$ and bridging them is a smooth quadratic curve. When the fixed point enters the region $x \in [-\epsilon, \epsilon]$ the bifurcations that follow are similar to as those observed in smooth maps. An example of a smooth bifurcation is the period doubling route to chaos. Where ϵ, ν, a are the smoothness, dissipation and nonlinearity parameters respectively. d is the dimension of the map while $k < d$ decides the number of positive lyapunov exponents in the system.

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