Periodic observation of a quantum system

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In this work we consider a particle in a harmonic oscillator potential as a dynamical system, and seek to obtain the Poincaré observations. In this case the 'state' is given by the wavefunction, and the periodic observations, akin to stroboscopic sampling, will yield a function mapping on to another function.

The usual interpretation of the wavefunction is that, if there is an ensemble of identically prepared systems and we do measurements of an observable in each one, we get the distribution given by $|\Psi|^2$. In our case we are asking what will be observed if we make repeated observations on a *single* system. Here our observable is position of the quantum system.

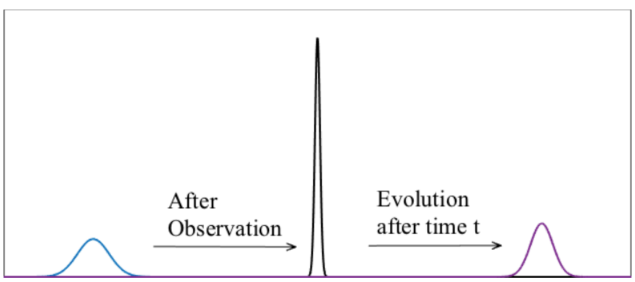

Figure 1: Illustration of the collapse of the wave function and its subsequent evolution.

We start from the ground state wave function. When we make an observation, the particle may be found anywhere where the value of Ψ is non-zero. After the observation, the wave function collapses to that point (which we take as a Gaussian function with a very narrow spread). This function then spreads following the Schrödinger equation. After a passage of time, we again make an observation. This process continues. The evolution of the wave function between two observations is illustrated in Fig. 1.

The harmonic oscillator has a characteristic time $t = 2\pi/\omega$. Now we ask: What will be the distribution for repeated observations at equal intervals.

The statistical distribution of the position for observation at intervals of $t/2$, $t/4$ and $t/\sqrt{2}$ are shown in Fig 3.

For simulating the Time-Dependent Schrödinger equation, we used the Crank- Nicolson Algorithm and then solved the tridiagonal system using guassian elimination.

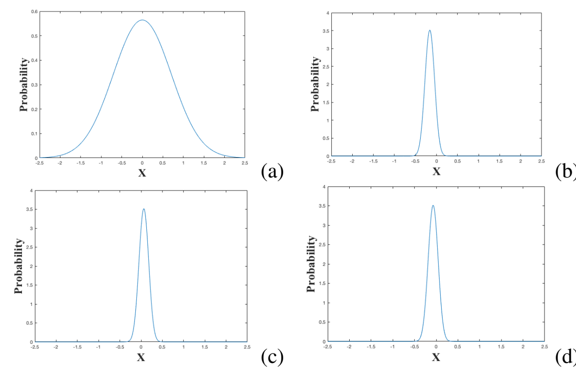

Figure 2: The wave functions at consecutive Poincaré observations at intervals of $t/2$.

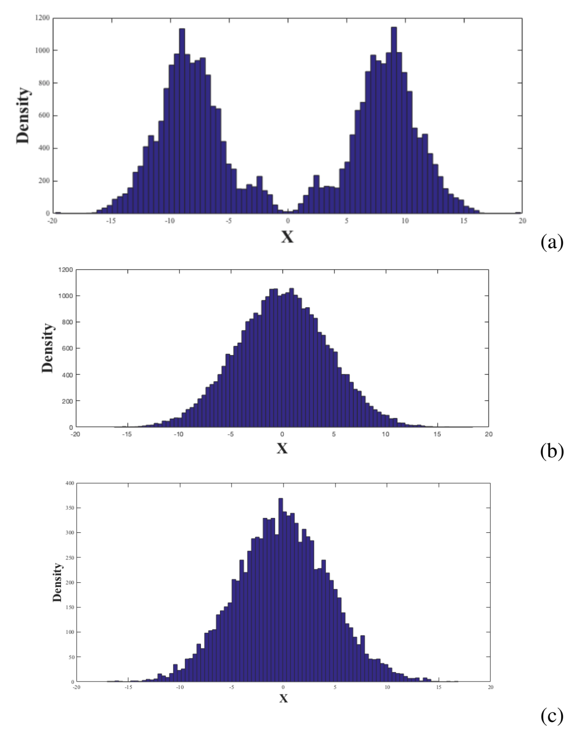

Figure 3: Histogram after observations: (a) at intervals of $t/2$, (b) at intervals of $t/4$, and (c) at intervals of $t/\sqrt{2}$.

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