Detecting Dynamical States Using Bicoherence Function

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One of the major limiting factors in deriving meaningful information from time series data is the presence of noise. Conventional quantifiers like the power spectrum, correlation dimension (D_2) and Lyapunov exponents are all known to be affected due to noise. It is in this context that we propose to use the bicoherence function to detect dynamical states from a noisy time series.

The bispectrum is defined as the Fourier transform of the triple auto-correlation function. When appropriately normalized, we have the bicoherence function which takes values between 0 and 1. Unlike the power spectrum, the bicoherence can be immediately seen to be not phase blind. This implies that it does not differentiate between different mean zero, symmetrically distributed noises, and hence does not need the method of Fourier phase randomized surrogates to check non-linearity[1].

It is well known that noise introduces new frequencies into the power spectrum. We point out that a Rössler limit cycle evolved with noise has a power spectrum and D_2 similar to a chaotic Rössler. It is then difficult to detect the underlying dynamical state by relying on these quantifiers alone. We show that the bicoherence function shows a distinct difference between a limit cycle evolved with noise and chaos. Hence the bicoherence function can be used to detect the true peaks in a power spectrum, when it is derived from a noisy time series.

We also exploit the bicoherence function to successfully distinguish noisy quasi-periodicity from strange non chaos. One of the popular methods to detect strange non chaos is the spectral scaling of peaks[2]. This method strobes the time series along the primary period, and counts the number of peaks above a threshold in the power spectrum. A power law scaling of peaks with threshold power, is often taken as an implication of strange non chaos. We show with the doubly driven pendulum as our test system, that a quasi-periodic time-series contaminated with red noise shows scaling similar to strange non chaos. To remove this ambiguity we propose looking at the main peak bicoherence(b_F(f)), defined as the bicoherence along the maximal power spectral peak. We count the peaks during spectral scaling, only if the b_F(f) is significant. We show that the power law scaling behavior persists for strange non chaos while it does not, for noise contaminated quasi-periodicity, as shown in Figure 1.

We apply these techniques to real world data, of variable star light curves showing period doubling behavior and strange non chaos[3, 4]. We point out that the many additional minor peaks in the power spectrum of the stars showing period doubling are actually of dynamical origin. For the strange non chaotic stars, we show that two of the stars can be confirmed as having strange non chaotic behavior.

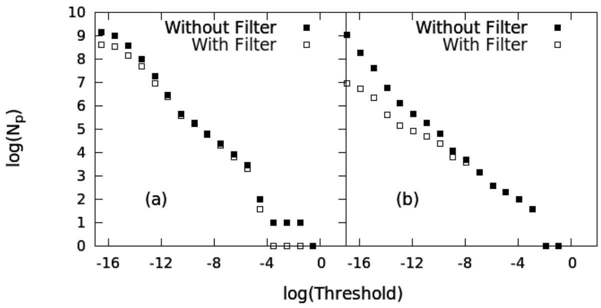

Figure 1: Spectral scaling of peaks for (a)strange non chaos case without and with the bicoherence filter (b) quasi-periodicity with added red noise without and with a bicoherence filter.

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