On the nonreciprocal nature of $\mathcal{PT}$ symmetrically coupled systems

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Recently, considerable interest has been shown in investigating systems which do not show parity ($\mathcal{P}$) and time-reversal ($\mathcal{T}$) symmetries separately but which exhibit a combined $\mathcal{PT}$ symmetry. In the literature, these $\mathcal{PT}$ symmetric systems were constructed by coupling a system with loss to a system with equal amount of gain [1]. We consider an interesting class of optical systems in which the interaction or coupling makes the systems $\mathcal{PT}$ symmetric. In this poster presentation, we discuss the dynamics of these $\mathcal{PT}$ coupled systems. We focus our attention on the non-reciprocal nature of these $\mathcal{PT}$-symmetric systems which have promising applications in constructing unidirectional devices like optical diodes [1].

The model that we consider is of the form

$$\begin{aligned} i\frac{d\phi_1}{dz} &= ia\phi_2 - k\phi_2 + \beta|\phi_1|^2\phi_1 + \alpha G(\phi_1,\phi_2,\phi_1^*,\phi_2^*), \\ i\frac{d\phi_2}{dz} &= -ia\phi_1 - k\phi_1 + \beta|\phi_2|^2\phi_2 + \alpha G(\phi_2,\phi_1,\phi_2^*,\phi_1^*). \quad (1) \end{aligned}$$

The model (1) represents a coupled pair of waveguides in which $\phi_1(z)$ and $\phi_2(z)$, respectively represent the complex amplitudes of light in the two waveguides with respect to the propagation distance z. In Eq. (1), the first term on the right hand side denotes the $\mathcal{PT}$ coupling. One can observe such $\mathcal{PT}$-symmetric coupling in the coupled waveguides of magneto-optic materials [2]. The parameters k and β represent the strength of evanescent field coupling and self-trapping nonlinearity respectively and $G(\phi_1,\phi_2,\phi_1^*,\phi_2^*) = -G(-\phi_1,-\phi_2,-\phi_1^*,-\phi_2^*)$ can be any odd order nonlinear term defining the nonlinear interactions. Eq. (1) is invariant under the combined operation of $\phi_1 \to -\phi_2$, $\phi_2 \to -\phi_1$, $i \to -i$ and $z \to -z$ and thus it is $\mathcal{PT}$ symmetric. In Eq. (1), there are no loss-gain terms. For certain forms of G, the system (1) can also be conservative, that is conservation of total power $|\phi_1|^2 + |\phi_2|^2$.

By considering different physically acceptable forms of G [3], we investigate whether the system exhibits a reciprocal dynamics or non-reciprocal dynamics where the reciprocal nature of the system represents the time reversibility and the non-reciprocal nature of the system arises due to the suppression in the time-reversibility nature. To determine whether the dynamics of the system is reciprocal or non-reciprocal, we consider two input situations, namely (i) $|\phi_1^{(1)}|^2(0) = 1$, $|\phi_2^{(1)}|^2(0) = 0$ and (ii) $|\phi_1^{(2)}|^2(0) = 0$, $|\phi_2^{(2)}|^2(0) = 1$. If the system is reciprocal in both the unbroken and broken $\mathcal{PT}$ regions then one can observe exact matching in the beam propagation patterns in the form $|\phi_1^{(i)}(z)|^2 = |\phi_2^{(j)}(z)|^2$, where $i,j \in 1,2$ and $i \neq j$. If the systems shows non-reciprocal dynamics then we find $|\phi_1^{(i)}(z)|^2 \neq |\phi_2^{(j)}(z)|^2$, where $i,j \in 1,2$ and $i \neq j$.

In the following, we summarize our results by excluding the nonlinearities of the form $G = \phi_2^{n+1}\phi_1^{*n}$. With different forms of the nonlinearity G (upto quinitic orders), we observe that all non-conservative cases are found to be non-reciprocal. Considering the conservative cases, the systems having nonlinearities of the form $G = |\phi_2|^2\phi_1^2\phi_2^*$ and $|\phi_1|^2|\phi_2|^2\phi_2$ are found to be non-reciprocal and all the other cases are reciprocal. Importantly, in many of the situations the non-reciprocal nature is observed only in the presence of self-trapping nonlinearity and in all the cases, the unidirectional propagation of light is achieved only in the presence of self-trapping nonlinearity. This clearly denotes the importance of self-trapping nonlinearity in this type of $\mathcal{PT}$ symmetric systems.

We have also extended our studies by making the linear $\mathcal{PT}$ symmetric coupling in (1) be nonlinear. In the case of nonlinear $\mathcal{PT}$ coupling systems, we found that all the conservative and non-conservative systems show non-reciprocal dynamics in the presence of self-trapping nonlinearities.

We have also observed the bifurcations that lead to spontaneous symmetry breaking in different $\mathcal{PT}$ symmetrically coupled systems. We found that the $\mathcal{PT}$ symmetric systems showing spontaneous symmetry breaking through tangent like bifurcation are found to be non-reciprocal and helpful in the construction of unidirectional transport devices. The systems that show symmetry breaking through pitchfork like bifurcation are found to be reciprocal (except in the cases with nonlinear interaction of the form $\phi_2^{n+1}\phi_1^{*n}$ with $n = 1,2$).

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References

[1] H. Ramezani, T. Kottos, R. El-Ganainy, and D. N. Christodoulides, Unidirectional nonlinear $\mathcal{PT}$-symmetric optical structures, *Phys. Rev. A*, 82:043803, 2010.

[2] A. Yariv, *Quantum Electronics*, John Wiley & sons, 1987.

[3] I. V. Barashenkov, D. E. Pelinovsky and P. Dubard, Dimer with gain and loss: Integrability and $\mathcal{PT}$-symmetry restoration, *J. Phys. A: Math. Theor.*, 48:325201 2015.

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