Generation of directed scale-free networks with tunable clustering and degree correlations

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With the growing importance of networks as a tool for studying complex systems, models of network generation have gained an importance of their own. In this project, we propose a model for generation of scale-free directed networks with tunable exponents in both the in and out degree distributions. We extend this model, by resorting to either the relevant degree preserving rewiring mechanisms or by introduction of additional links between existing nodes, to tune the clustering coefficients and degree correlations to desired values. We monitor the network, during growth and rewiring, by calculating the relevant measures. We also study the role of average degree and the scaling with network size.

Most of the models proposed for generation of scale-free directed networks (except [1, 2, 3]) suffer from a drawback, that the out-degree distribution is either constant or decays as a poissonian or exponential distribution. This is in contradiction with observations from real networks, where the out-degree distribution is a power-law. In the first part of our work, we introduce a modification of the price model[4] that results in power-laws in both the in and out degree distributions, with the exponents tunable between 2 and 3. The construction process in the price model is modified so that, at every step, the new node forms c_{in} in-links from existing nodes, chosen randomly with a probability proportional to their out-degrees. This is done in addition to the c_{out} out-links formed from the node to existing nodes. This results in a directed network with scale-free nature in both in and out degree distributions. The exponents of these distributions can be tuned by varying the values of c_{in} and c_{out}.

In the second part of our work, we extend the modified price model to tune the amount of clustering in the network by the following 2 methods: The first is the mechanism of Degree Preserving Rewiring (DPR). We generate a network using the extended price model and measure its initial clustering coefficient. We then rewire the links, while preserving the degrees of the nodes, to close open triplets in the network. The rewiring is retained only if it improves the clustering coefficient and reverted otherwise. The directions of the rewired links can be chosen such that only the desired clustering coefficient is modified. The amount of clustering is tuned using the number of rewirings. The second method involves addition of extra links between existing nodes. While this can be done randomly, preferentially or by triadic closure, we go with preferential addition. We modify the extended price model to add c_{add} additional links between existing nodes. The source nodes for these edges are chosen randomly with a probability proportional to the out-degrees and the target nodes are chosen with probability proportional to the in-degrees. The clustering is tuned by changing the value of c_{add}. The above 2 methods result in clustered scale-free directed networks. Other properties of the network, like degree correlations and the average path length, are also monitored during the process of growth and rewiring.

In the third part of our work, we present a model to generate scale-free directed networks with tunable degree correlations. This model is similar to the DPR method presented in the case of networks with tunable clustering, although the mechanism of rewiring is different. Two randomly chosen links are rewired between nodes of similar/dissimilar degrees to generate assortative/dissortative networks, while preserving the degrees of the nodes. Once again, the rewiring is retained only if it contributes to the desired change and reverted otherwise. In all cases, the DPR is done in such a way that no multi-edges and self-loops are created. The amount of correlation in the network is tuned by changing the number of rewirings.

References

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