Controlling optical similaritons in tapered graded-index waveguide in presence of $\mathcal{PT}$ symmetric potential

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In recent years, there has been considerable interest experimentally as well as theoretically in the use of tapered graded-index nonlinear waveguide in optical communication systems [1]. Tapering effect tends to reduce reflection losses and mode mismatch [2] which results in improving the coupling efficiency between fibres and waveguides. The concept of $\mathcal{PT}$ symmetry in optics was first introduced by Musslimani et al. [3] in 2008. The proposed $\mathcal{PT}$ symmetry can be implemented by a judicious inclusion of combination of gain and loss regions and process of index guiding in optical waveguides. This is modeled by complex refractive index distribution $n = n_0 + n_R(z, x) + in_I(z, x)$ which plays the role of optical potential. It is suggested that real part (index waveguiding profile) $n_R(z, x)$ should be symmetric and imaginary part $n_I(z, x)$ should be antisymmetric in the transverse direction.

Recently, authors [4, 5] have presented a theoretical technique, to control the intensity and widths of self-similar waves, by invoking the isospectral Hamiltonian approach introduced by Infeld and Hull [6] and Mielnik [7]. While reducing generalized nonlinear Schrödinger equation (GNLSE) to standard NLSE to obtain self-similar solutions, the constraint equation for tapering function resembles Schrödinger equation in quantum mechanics (QM), with tapering function as potential and width function as ground-state wavefunction. Thus, employing isospectral Hamiltonian approach from supersymmetric quantum mechanics (SUSY QM) [8] in tapered graded-index waveguide, we can generate a class of tapering function $F(Z)$ and width function $W(Z)$ by introducing Riccati parameter.

We are considering tapered graded-index waveguide whose refractive index is $n = n_0 + n_0[n_R(z, x) + in_I(z, x)] + n_1F(z)x^2 + n_2\gamma(z)I(z, x)$ where x and z stand for dimensionless spatial co-ordinate and the propagation distance respectively. The shape of the taper $F(z)$ can be modelled appropriately depending upon the practical requirements. The beam propagation in tapered graded-index waveguide is governed by following generalized nonlinear Schrödinger equation (GNLSE)

$$iu_z + \frac{1}{2}u_{xx} + \gamma(z)|u|^2u + F(z)x^2u + [v_1(x, z) + iw_1(x, z)]u = 0, \quad (1)$$

The refractive-index potential v_1 is assumed to be even, while the function w_1, assumed to be odd, stands for the gain-loss landscape profile. Using following gauge and similarity transformation

$$u(x, z) = A(z)U(X, Z)e^{i\theta(x,z)} \quad (2)$$

where $X = \frac{x}{w(z)}$ and $Z = Z(z)$ we reduce Eq.(1) to following constant-coefficient NLSE

$$iU_Z + \frac{1}{2}U_{XX} + G_3|U|^2U + [V(X) + iW(X)]U = 0, \quad (3)$$

where G_3 is constant. The effective propagation distance, amplitude and quadratic phase is given as $Z = \int_0^z \frac{1}{w^2(s)}ds$, $A(z) = \sqrt{\frac{G_3}{\gamma}}\frac{1}{w(z)}$ and $\theta(x, z) = \frac{w_z}{w(z)}\frac{x^2}{2}$ respectively. The parameter $w(z)$ represents the dimensionless width of the beam. Further, the constraints on real and imaginary parts of $\mathcal{PT}$ symmetric potential are given by $v_1(x, z) = Z_zV(X)$ and $w_1(x, z) = Z_zW(X)$ respectively. For Scarf II type potential $V(X) = V_0\mathrm{sech}^2(X)$ and $W(X) = W_0\mathrm{sech}(X)\tanh(X)$ where V_0 and W_0 are arbitrary constants, Eq.(3) possesses following soliton solution

$$U(X, Z) = \sqrt{\frac{18 + W_0^2 - 18V_0}{18G_3}}\mathrm{sech}(X)e^{i\phi(X,Z)}, \quad (4)$$

with $\phi(X, Z) = Z(z)/2 + (W_0/3)\tan^{-1}[\sinh(X)]$. Further, the constraint equation for tapering

$$\frac{d^2w}{dz^2} - F(z)w(z) = 0, \quad (5)$$

resembles Schrödinger equation in quantum mechanics (QM), with tapering function as potential and width function as ground-state wavefunction. Thus, employing isospectral Hamiltonian approach from SUSYQM in tapered graded-index waveguide, we can generate a class of tapering function $F(z)$ and width function $w(z)$ by introducing Riccati parameter.

The general form of tapering for sech^2-type waveguide is $F(z) = n^2 - n(n + 1)\mathrm{sech}^2 z$ with width is given by $w(z) = \mathrm{sech}^n z$. For $n = 1$, a class of $\hat{w}(z)$ can be given as

$$\hat{w}(z) = \frac{\sqrt{c(c + 1)}\mathrm{sech}(z)}{c + (1 + \tanh(z))}, \quad (6)$$

where c is the Riccati parameter. It is clear from Figure(1) that Riccati parameter c has significant effect on the intensity and width of the optical similaritons of Eq.(1). For small values of Riccati parameter c, the intensity of similaritons increases significantly whereas the width of the similaritons decreases.

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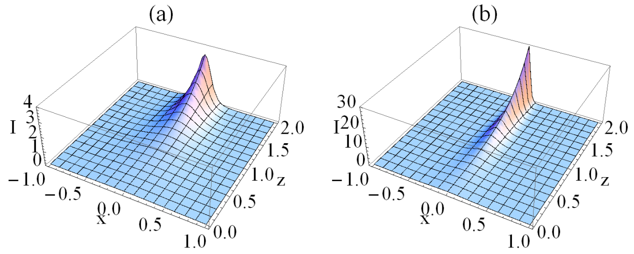

Figure 1: Intensity of similaritons for (a) $c = 1$ and (b) $c = 0.1$. The other parameters chosen are $\gamma(z) = 1,\ G_3 = 1,\ V_0 = W_0 = 1$.

In conclusion, we have obtained bright similariton solutions for GNLSE governing beam propagation in tapered graded-index waveguide in presence of $\mathcal{PT}$ symmetric potentials. Further, the constraint equation for tapering resembles the Schrödinger equation in QM enabling us to generate a class of tapering function $F(z)$ and width function $w(z)$ by introducing Riccati parameter which imposes significant effect on intensity and width of similaritons. Interestingly, the intensity of similaritons can be made very large, thus paving the way for experimental realization of highly energetic pulses.

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