Interplay of degree degree correlation and driven mechanism for cluster synchronization

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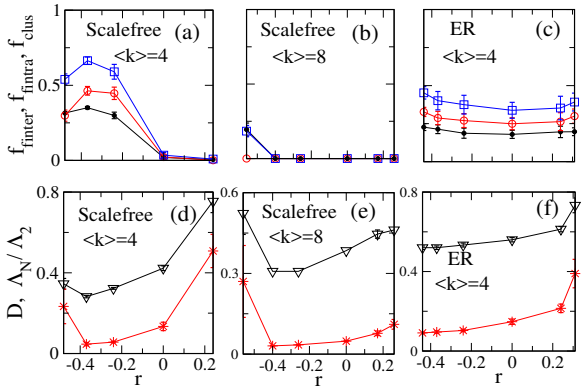

Figure 1: (a), (b) shows dependence of CS on r for scale free network of different $\langle k \rangle$ values. For ER netoworks CS remains almost independent of r (c). Various structural and dynamical measures such as intercluster connections (f_{inter}), intracluster connections (f_{intra}), CS (f_{clus}), laplacian eigenvalue ratio (Λ_N/Λ_2) and diameter (D) are represented by $\bullet$, $\circ$, $\square$, $*$ and $\bigtriangledown$, respectively. Here $N = 200$ for all the networks and each data point is obtained for average over 10 random realizations of the networks. Each variable in (d), (e), (f) is normalized by its maximum value

We study the evolution of coupled chaotic dynamics on networks and investigate the role of degree-degree correlation (r) on the networks cluster synchronizability (CS). We find that an increase in the disassortativity can lead to an increase or a decrease in CS (represented by f_{clus}) depending on the degree distribution and average connectivity ($\langle k \rangle$) of the network. Networks with the heterogeneous degree distribution exhibit significant changes in CS as well as in the phenomena behind CS as compared to those of the homogeneous networks. Interestingly, CS of a network may be very different from global synchronizability due to the presence of the driven phenomenon behind the cluster formation. Global synchronizability of a network can be defined using a ratio of the largest to the first non-zero eigenvalue of the corresponding Laplacian matrix as Λ_N/Λ_2, where $\Lambda_N > \Lambda_{N-1} > \ldots > \Lambda_2 > 0$ are eigenvalues of the Laplacian matrix ($L = \mathcal{D} - A$), with $\mathcal{D}$ being the matrix having degree of nodes at the diagonal elements and all other off-diagonal elements being zero. Scale free disassortative networks shows decrease and increase in CS as $\langle k \rangle$ increases whereas global synchronizability decreases continously. Furthermore, we show how degeneracy at the zero eigenvalues provides an understanding to the occurrence of the driven phenomenon behind the synchronization in disassortative networks.

The results demonstrate an importance of degree-degree correlations in determining cluster synchronization behaviour of complex networks, and hence have potential applications in understanding and predicting dynamical behaviour of complex systems ranging from brain to social systems.

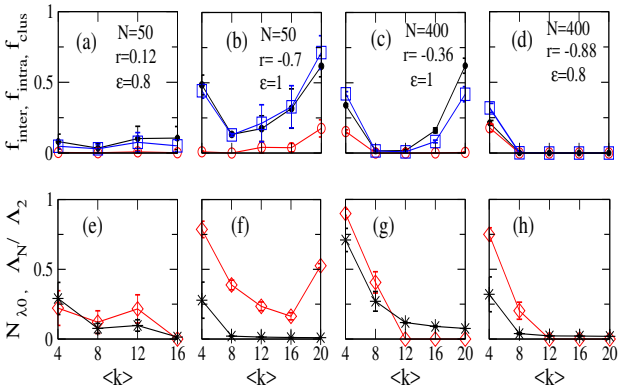

Figure 2: Various dynamical and structural properties are plotted as a function of average degree $\langle k \rangle$. (a), (b) and (c) plot f_{clus} ($\square$), f_{inter} ($\bullet$) and f_{intra} ($\circ$) for SF networks of various size and degree-degree correlations. (e), (f) and (g) plot Λ_{max}/Λ_2 ($*$) and degeneracy at the zero eigenvalue of the adjacency matrix (N_{λ_0} ($\diamond$)) for the same network parameters as of the corresponding top subfigures. (d), (h) plots behaviour for ER networks.

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References

[1] Sarika Jalan, Anil Kumar, Alexey Zaikin and Jürgen Kurths, *Interplay of degree degree correlation and driven mechanism for cluster synchronization* (under review)