The Duffing - Holmes Oscillator: A Theoretical Analysis of the Magneto-elastic Interactions

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The magneto-elastically buckled beam, also known as the Duffing - Holmes oscillator is a classic example of a nonlinear oscillator that exhibits chaotic motions. This system serves as a model to analyze the motion of elastic structures in magnetic fields. The magnetoelastic oscillator consists of a ferromagnetic cantilever beam bent between two external magnets and undergoing forced oscillations, as shown in Fig. 1. The interaction between the magnets and the ferromagnetic beam is nonlinear and depends on the dimensions and the field strength of the magnets, and the distance between the magnets.

In their seminal work [1], Moon and Holmes elucidated that the system has a sixth order magneto-elastic potential and can have either one, three or five static equilibrium positions [1]. The oscillator is said to be monostable, bistable or tristable when it has one, two or three stable equilibrium positions respectively.

Since then, experiments and mathematical models based on data fit from experiments are abound in literature that describe the dynamics of the oscillator. These models generally utilize the non-dimensional form of equations corresponding to a forced oscillator with nonlinear restoring forces of the form $f_r(x) = ax + bx^3 + cx^5$. The stiffness coefficients a, b and c are derived either from experiments [2] or from numerical simulations [3]. Under such cases, it becomes difficult to trace the effect of change in a physical parameter, say, the distance between the magnets, on the behavior of the system. An analytical model, relating the physical parameters to the system dynamics, would be helpful in such parametric studies.

Hence, the authors derived an analytical model that incorporates the dependence of magneto-elastic potential on parameters such as the dimensions and the field strength of the magnets, and the distance between the magnets [4]. Approximate closed form analytical expressions were derived for the stiffness coefficients a, b and c and reported in [4].

In the model described in [4], it is considered that both the base magnets have equal field strength and are placed symmetrically at the bottom. In this work, the model from [4] is extended to include the disparities in the field strength of the two base magnets and an offset in their position. This would break the symmetry in the magnetic field and lead to a restoring force of the form $f_r(x) = k_1x + k_2x^2 + k_3x^3 + k_4x^4 + k_5x^5$.

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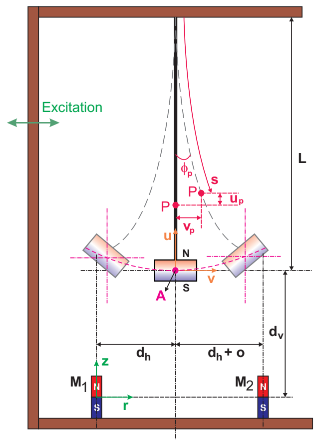

Figure 1: Schematic representation of the magneto-elastic oscillator.

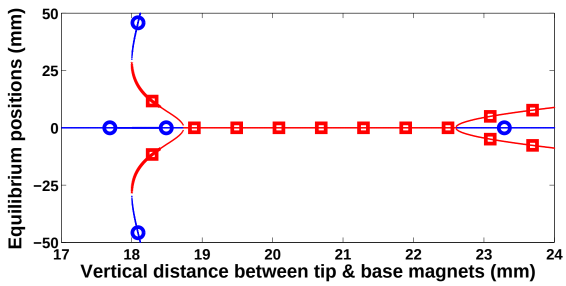

Figure 2: Bifurcation of static equilibrium positions w.r.t. vertical distance between the base and tip magnets.

Closed form expressions have been derived for the new stiffness constants k_1 to k_5. The developed analytical model has been numerically simulated to obtain the bifurcation of static equilibrium positions with respect to the offset and disparities in the field strength. The bifurcation of static equilibrium positions with respect to vertical distance between the base and tip magnets d_v is shown in 2. Other simulated results along with the derived expressions will be reported in the final manuscript.

References

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