Interaction of Neimark-Sacker and period doubling bifurcations in a vibro-impact system

Ananya Mondal* and Soumitro Banerjee *

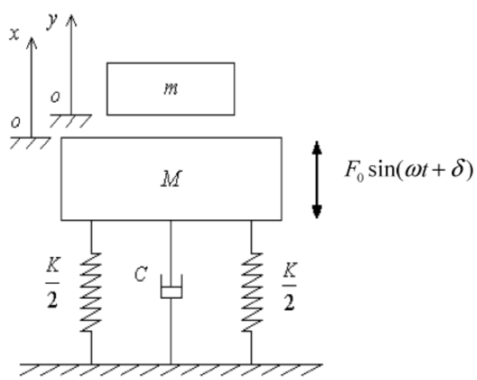

Figure 1: Vibro-impact system

In this paper we investigate the bifurcation from an invariant closed curve (a torus in continuous-time system) to two disjoint closed curves [1]. We take a mechanical vibro-impact system as a specific example in which such a bifurcation has been reported [2].

Two possible mechanisms of such a bifurcation have been proposed earlier : (a) a period doubling bifurcation followed by a Neimark-Sacker bifurcation, and (b) a Neimark-Sacker bifurcation followed by a period doubling of the unstable fixed point [2]. A recent paper has proposed a different mechanism of this bifurcation in terms of Lyapunov bundles [3]. In this paper we seek to resolve the issue.

System description: Vibro-impact systems are often encountered in practice, for instance, in the models of hammer-like devices, rotor-casing dynamical systems, collisions of solids etc. These systems are good testing bench for nonlinear theories. Figure 1 shows the model of the vibro-impacting system. The mass M is connected to a linear spring with a stiffness K and a damper C. The excitation on M is $F_0 \sin(\omega t + \delta)$. The mass M impacts mass *m* whenever $x = y$ and the relative velocity is non-zero. After impacting, *m* becomes a free body and moves in the field of gravity g and M becomes a 1-degree of freedom forced oscillator. The dynamical equations for the two masses in between impacts are:

$$M\ddot{x} + C\dot{x} + Kx = F_0 sin(\omega t + \delta), \tag{1}$$

$$\ddot{y} = -g. \tag{2}$$

The impacting conditions follow from the conservation of momentum and coefficient of restitution as,

$$\dot{x}_+ = \frac{1-\mu R}{1+\mu}\dot{x}_- + \frac{\mu(1+R)}{1+\mu}\dot{y}_- \tag{3}$$

$$\dot{y}_+ = \frac{1+R}{1+\mu}\dot{x}_- + \frac{\mu-R}{1+\mu}\dot{y}_- \tag{4}$$

where $\dot{x}_-$, $\dot{y}_-$ are the approach velocities and $\dot{x}_+$, $\dot{y}_+$ are the departure velocities of the masses at the instant of impact, $\mu = m/M$ and R is the coefficient of restitution.

Using these equations the behaviour of the system has been studied at different bifurcation parameters. We have found that the system demonstrates torus doubling resulting in two disjoint loops. The eigenvalues of the relevant fixed points before and after the bifurcation have been computed using an algorithm recently developed in our group.

The paper presents our results in this aspect which leads towards the formulation of a universal theory underlying such a phenomenon.

References

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[2] Ding, W. C., Xie, J. H., and Sun, Q. G., *Interaction of Hopf and period doubling bifurcations of a vibro-impact system*, Journal of Sound and Vibration, 275(1), 27-45, 2004.

[3] Kamiyama, K., Komuro, M., Endo, T., Aihara, K., *Classification of bifurcations of quasi-periodic solutions using lyapunov bundles*, International Journal of Bifurcation and Chaos, 24(12), 1430034, 2014.

*Department of Physical Sciences, Indian Institute of Science Education and Research (IISER) Kolkata , email:am12ms029@iiserkol.ac.in