Decoupled Terminal Sliding Mode Control for Stabilization of Double Inverted Pendulum

Sudhir Raj *

Abstract: Decoupled Sliding mode control has received much attention due to its major advantages such as guaranteed stability, robustness against parameter variations, fast convergence and simplicity in implementation and therefore has been widely used to control nonlinear underactuated systems. Underactuated mechanical system control is a difficult task in nonlinear control. Decoupled Terminal sliding mode control decouples the whole system into second order systems such that each subsystem has a seperate control objective expressed in terms of hybrid sliding surface. This method eliminates the use of fuzzy rules without any decrease in system performance. Double Inverted Pendulum mounted on a cart was simulated with this method to achieve pole Stabilization. Simulation results verify the effectiveness of Decoupled Terminal Sliding Mode Control with hybrid surfaces.

Keywords: Hybrid Sliding Mode Control, Decoupled Sliding mode Control, Double Inverted Pendulum.

1. Introduction

Sliding mode control (SMC) has been considered as an effective tool in different studies for control systems. The robustness of SMC against the disturbances and model uncertainities makes the system stable. However, the chattering phenomenon degrades the system performances, it may even lead to instability. Second order sliding mode , the twisting and super-twisting algorithm are used to solve the tracking control of underactuated manipulators.Terminal sliding mode control was proposed for second order nonlinear uncertain systems. By using function augmented sliding hyperplane, it is guaranteed that the output tracking error converges to zero in finite time which can be set arbitrarily. The proposed scheme eliminates the reaching phase problem so that the closed loop system always shows the invariance property to parameter uncertainties. Decoupled terminal sliding mode control with hybrid sliding surfaces [1] was proposed for a class of underactuated systems and it provides a easy way to decouple fourth order systems into two second order subsystems. The auxiliary subsystem can be incorporated into the main one via a two level decoupling strategy. Decoupled fuzzy Sliding mode control [2] was proposed for a class of fourth order nonlinear systems with only five fuzzy control rules. Bayramoglu and Komurcugil [3] propose a nonsingular Decoupled terminal sliding control method to avoid singularity and convergence of the two subsystems to their equilibrium points can be achieved. The mechanical systems with fewer controls than the number of system variables to be controlled are known as underactuated systems. Underactuated systems have wide applications in space robots, underwater robots, Pendubot , Acrobat, overhead crane and cart–pole.

Double Inverted Pendulum is an underactuated system and it is a difficult task in control problems. The whole system is decoupled into three subsystems A, B and C. The subsystem A is formed by $\theta_1, \dot{\theta}_1$ and with sliding surfaces s_1. The subsystem B is formed by $\theta_2, \dot{\theta}_2$ and with sliding surfaces s_2. The subsystem C is formed by $x, \dot{x}$ and with sliding surfaces s_3. The control objective is twofold: to keep the angle of both poles close to zero in an upright position and to allow the cart to move freely without any boundary limits. The paper is organized as follows. In Section 2, Modelling of Double Inverted Pendulum has been done. In section 3, Decoupled terminal Sliding mode Controller is described for Stabilization of Double Inverted Pendulum mounted on a cart. Simulation results are given in Section 4. Finally, the conclusions are addressed in Section 5.

2. Modelling of Double Inverted Pendulum

The structure of Double Inverted Pendulum system is shown in Fig.1. It is subjected to horizontal force u. Pole 1 is the pole connected to the cart and pole 2 is the one above pole 1. The displacement of the cart is denoted by x.

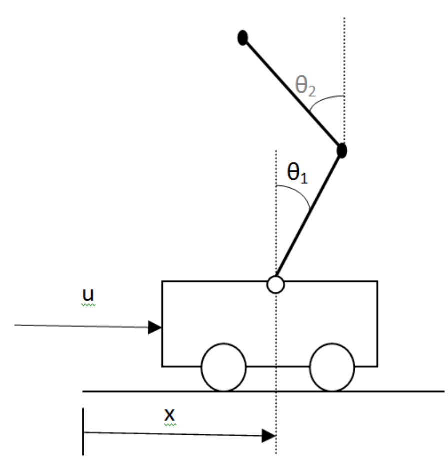

Figure 1: Double Inverted Pendulum mounted on a cart

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$$\dot{x}_1 = x_2 \quad (1)$$
$$\dot{x}_2 = f_1 + b_1 u + d \quad (2)$$
$$\dot{x}_3 = x_4 \quad (3)$$
$$\dot{x}_4 = f_2 + b_2 u + d \quad (4)$$
$$\dot{x}_5 = x_6 \quad (5)$$
$$\dot{x}_6 = f_3 + b_3 u \quad (6)$$

where

$$x_1 = \theta_1, x_2 = \dot{\theta}_1, x_3 = \theta_2, x_4 = \dot{\theta}_2, , x_5 = x, x_6 = \dot{x}$$

are the states of the Double Inverted Pendulum mounted on a cart. The system dynamics of Double Inverted Pendulum are descibed by the equations (1- 6).

3. Decoupled Sliding Mode Control

The sliding surfaces are taken as s_1& s_2 respectively. Z represents a decaying oscillating signal whose value is calculated by equation no. 9. The control input to the Double Inverted Pendulum mounted on a cart is given by Equation no. 10.

$$s_2 = \dot{\theta}_2 + 4.2sign^{0.9}(\theta_2) + 1.7\theta_2 \quad (7)$$
$$s_1 = \dot{\theta}_1 + 5sign^{1.1}(\theta_1 - Z) + sign^{1.43}(\theta_1 - Z) \quad (8)$$
$$Z = sat(s2, 10) \quad (9)$$
$$u = -(f_1 + (|\theta_1 - Z|^{1.33} + |\theta_1 - Z|^{0.4})(\dot{\theta}_1 - \dot{Z}) + 2s_1 + 4|s_1|^{0.714} sign(s_1))/b_1 - 5sat(s_1 b_1, 1) \quad (10)$$

4. Simulation Results

Double Inverted Pendulum was simulated by Decoupled Terminal Sliding Mode Control with hybrid surfaces (FDTSMC). Fig 2 shows the variation of θ_1& Z vs time in Sliding Mode Control. Fig 3 shows the variation of $\theta_2$vs time in Sliding Mode Control. Fig. 4 shows the variation of Control input u vs time in Sliding Mode Control. Settling time for θ_1&$\theta_2$are 5 second using this method. In the simulation, the following parameters are used.

$l_1 = 1m$, $l_2 = 1m$, $g = 9.8m/sec^2$, $m_c = 1Kg$, $m_1 = 1Kg$, $m_2 = 1Kg$

5. Conclusions

Decoupled Sliding mode control with hybrid sliding surfaces has been used for Stabilization of Pole angles of Double Inverted Pendulum mounted on a cart. Hybrid sliding surface are used to achieve finite time convergence of the System states. Simulation results show the effectiveness of this method. Decoupled Sliding Mode control with hybrid surfaces eliminates the need of Fuzzy logic controller in balancing the pole angles of Double Inverted Pendulum mounted on a cart. This method shows its effectiveness in non linear control of underactuated systems.

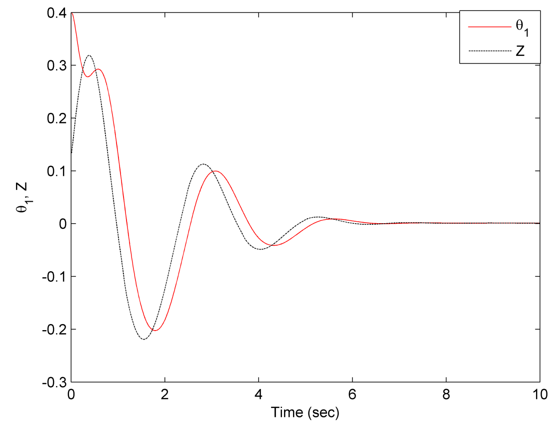

Figure 2: Plot of θ_1, Z vs Time in FDTSMC

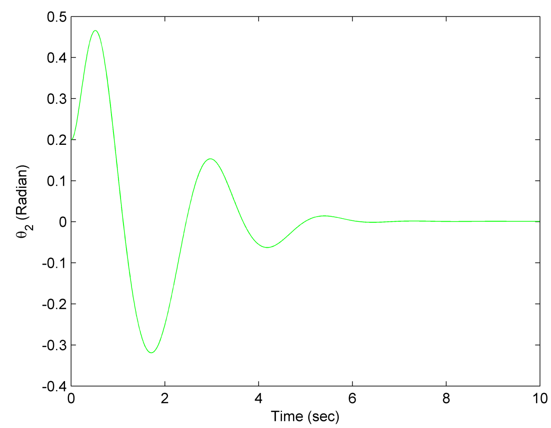

Figure 3: Plot of $\theta_2 vs$ Time in FDTSMC

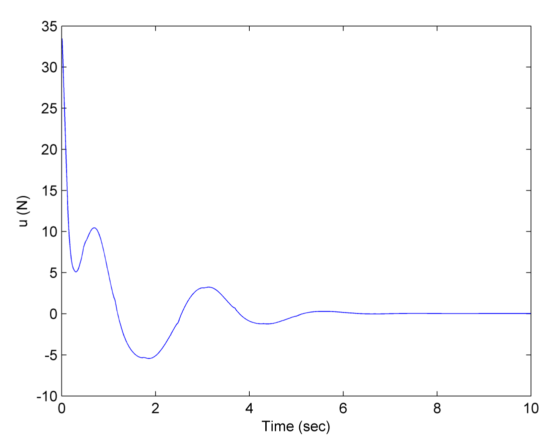

Figure 4: Plot of Control input u vs time in FDTSMC

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