Construction of new rational potentials and their solutions using Darboux transformations

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The discovery of exceptional orthogonal polynomials (EOPs) by Kamran *et.al.,* [1], [2] extended the class of orthogonal polynomials arising as the solutions of the Sturm-Liouville operator. The first polynomials to be constructed were the exceptional Laguerre and Jacobi polynomials of codimension 1. Later on polynomials with codimension m have also been constructed. The characteristic feature of this new class of polynomials is that the series can start with a polynomial of degree $l, (l = 1, 2, ...)$ and still forms a complete set with respect to a positive-definite measure. Another feature of these polynomials is that their second order differential operators have regular singular points (read poles) in their coefficients. The discovery of these new polynomial classes also extends the scope of the Bochner theorem on characterization of orthogonal polynomial systems defined by Sturm-Liouville's problem [1], [3].

The connection between the EOPs and shape invariant potentials (SIPs) was first realized by Quesne [4]. Using Darboux transformation, she constructed new potentials, rational extensions of the radial oscillator and the Scarf potential, with X_1 exceptional Laguerre and Jacobi polynomials as solutions. Later, Sasaki *et.al.,* constructed potentials which have solutions in terms of EOPs of higher codimension [5]. In our recent study, we have shown that the QHJF in combination with supersymmetric quantum mechanics (SUSYQM) provides a simpler method to construct these new rational potentials [6]. The Riccati equation plays an important role here and the connection between the Riccati equation and second order linear differential equation is utilized to arrive at the results. The method also exploits supersymmetric quantum mechanics techniques. Our method allows one to construct a hierarchy of generalized exactly solvable rational potentials and their solutions in simpler way. In addition, the method automatically takes care of the weight regularity problem, which arises when one rationalizes the weight function.

In the current study, we explore the idea of further generalizing this method to construct new rational potentials. Though the method is general and applicable to all ES models, we have specifically used it to study the rational radial oscillator potential discussed in [4], [5] and [6]. We have constructed rational extensions to this potential. The preliminary results for the solutions of these new potentials look very promising and offer an interesting prospect of either extending the class of EOPs or leading us to a totally different class of polynomials. This work is still under progress and the results will be presented.

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