Microtransitions in 2-D Load Bearing Hierarchical Networks

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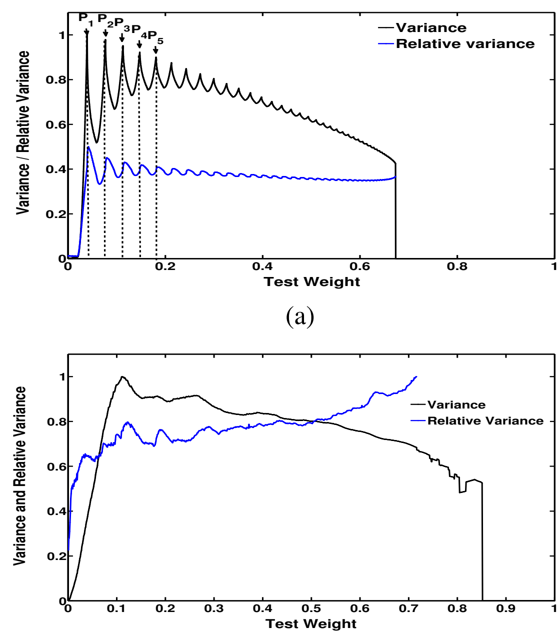
The prediction of the critical point of a transition has been a topic of great interest in research. Hence precursors of critical point need to be identified. Recent model studies have shown that microtransitions act as precursors to the percolation transition, a transition to large scale connectedness of network on the addition of links. It has been seen that the relative variance of the order parameter shows sharp peaks before the transition point for both the continuous as well as discontinuous percolation models [1]. These peaks arise because of change in order parameter in microscopic scale and are known as microtransitions. The relative positions of microtransitions obey a scaling law, which gives the parameter value corresponding to the critical point.

We investigate the microtransitions in avalanche transmission on 2-D load bearing hierarchical networks. The 2-D load bearing hierarchical network is the q(0,1) case of the Coppersmith model [2]. It is identical to the river network model [3] and also similar to the Voter model [4], and the models of lungs inflation [5]. We have defined an avalanche as a process of weight transmission on the network. When a weight is deposited to a site in the first layer it retains a weight equal to its capacity and transmits the rest to the connected site in the layer below. Thus the weight is transmitted in the downward direction. If there is still excess weight left at the bottom most layer of the network it is then transmitted to a randomly chosen site of the first layer. In this manner, if the weight encounters a fully saturated site, and also has no alternate path to take, then the transmission is considered to have failed. If the transmitted weight is absorbed at some site in the network then the transmission corresponds to a successful transmission. The order parameter is defined as the fraction of successful transmission for a given test weight. It varies continuously as a function of test weight for a typical realisation of the network, the original lattice and shows a discontinuous variation with test weight for the critical realisation of the network , the V lattice [6]. We see sharp peaks in the variance of the order parameter before the transition point for the original lattice as well as for the V lattice Fig. [1]. The variance is defined as;

$$W_{variance} = \overline{O^2} - \overline{O}^2$$

where, O is the total absorbed weight by the network for some given test weight. The average is taken over the total number of nodes, which are fully occupied. The relative positions of the microtransitions obey a scaling law. From



(a)

(b)

Figure 1: Microtransition cascades (a) for the V lattice of size 50×50 (b) for the original lattice of size 50×50 and averaged over 20 realisations.

the scaling relation we calculate the critical point, which is in good agreement with the value of the critical point from the order parameter plot for the V lattice case. We also analyse the correlation function and the structure factor for the V lattice.

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References

[1] Wei Chen, Malte Schröder, Raissa M. D'Souza, Didier Sornette, and Jan Nagler. Microtransition cascades to percolation. *Physical Review Letters*, 112(15):1–5, 2014.

[2] S. Coppersmith, C. Liu, S. Majumdar, O. Narayan, and T. Witten. Model for force fluctuations in bead packs. *Physical Review E*, 53(5):4673–4685, 1996.

[3] A. E. SCHEIDEGGER. A Stochastic Model for Drainage Patterns Into an Intramontane Treinch. *International Association of Scientific Hydrology. Bulletin*, 12(1):15–20, 1967.

[4] David Griffeath. *Additive and Cancellative Interacting Particle Systems*. Springer-Verlag Berlin Heidelberg, 1st edition, 1979.

[5] B Suki, J S Andrade, M F Coughlin, D Stamenović, H E Stanley, M Sujeer, and S Zapperi. Mathematical modeling of the first inflation of degassed lungs. *Annals of biomedical engineering*, 26(4):608–17, 1998.

[6] Ajay Deep Kachhvah and Neelima Gupte. Transmission of packets on a hierarchical network: Statistics and explosive percolation. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, 86(2):1–12, 2012.