Dissipation dynamics with two finite chaotic environments

Diptanil Roy and A.V. Anil Kumar *

Dissipative systems are open thermodynamical systems which operate far from their thermodynamic equilibrium. They exchange energy and matter with their environment and the dynamics of the dissipation is governed chiefly by the interaction between the system and the environment and also by the nature of the environment. The environment is often modelled by infinite harmonic oscillators with continuous distribution of frequencies. However, in specific cases [1], it is more useful to choose environments with a few, finite chaotic degrees of freedom.

Marchiori and Aguiar [2] studied dissipation using a finite environment of N quartic oscillators with $2N$ degrees of freedom which was coupled to a single harmonic oscillator and obtained interesting results. We extend their study by looking at the dynamics of the dissipation when we introduce a second bath of N identical quartic systems different from the 1st bath. We look at the energy flow into the environment as a function of the chaotic parameters of the bath and also try to develop a linear response theory to describe the system. In our simulations, the central system is a harmonic oscillator with a single degree of freedom, whereas the environment has $2N+2N$ degrees of freedom. The quartic oscillators in the environment interact through the harmonic oscillator only, which is true for both the baths.

We consider a conservative system governed by the Hamiltonian

$$\begin{aligned} H &= H_{HO} + H_{E_1} + H_{E_2} + \lambda_N H_I \\ H_{HO} &= \frac{p^2}{2m} + \frac{m\omega_0^2 q^2}{2} \\ H_{E_i} &= \sum_{n=1}^{N} \left[\frac{p_{x_{n_i}}^2 + p_{y_{n_i}}^2}{2} + \frac{a_i}{4}(x_{n_i}^4 + y_{n_i}^4) + \frac{x_{n_i}^2 y_{n_i}^2}{2} \right] \\ H_I &= \sum_{n=1}^{N} q(x_{n_1} + x_{n_2}) \end{aligned}$$

The two baths are identified by the parameter a_i completely. The QS is highly chaotic for $a_i \le 0.1$. The equations of motion have been solved numerically using a fourth-order Runge-Kutta integrator [3].

We observe expected behaviour in the known regimes [1, 2]. The energy of the HO shows a clear exponential decay with very minor fluctuations when either of the baths is chaotic. There is a smooth transition from the chaotic regime to the integrable regime where the energy of the HO falls off exponentially first, then linearly and finally remains constant with minor fluctuations.

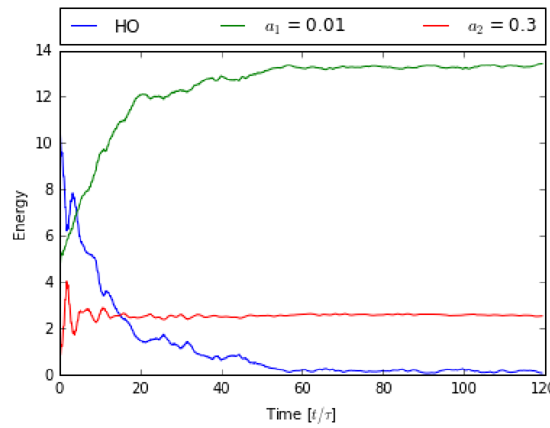

Figure 1: Energies of the **HO** and the two baths with different chaotic parameters and different initial energies. The time is expressed in terms of the time period of the harmonic oscillator $\tau = 20.93$. The initial energy of the HO is 10 and the two baths E_1 and E_2 have energies $0.05N$ and $0.01N$ respectively and $N = 100$.

However, we observe some interesting behaviour regarding the energy flow as shown in the plot above. The baths are characterised by their chaotic parameters and their initial energies only. From the initial energies, we can define temperatures for the baths using the general equipartition theorem which suggests that the chaotic bath E_1 has a higher temperature than bath E_2 for the realisation (1) above. We expected more energy to flow to the bath with lower temperature when it is driven towards equilibrium. However, as is clearly evident from the simulation, more than 90% of the energy flows to the chaotic bath, irrespective of the initial energies of the baths. This leads us to deduce that the energy flow is entropy driven. We try to explain this behaviour using linear response theory while discussing the utility of such an environment as a reservoir as discussed in [2].

References

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*Diptanil Roy is an Integrated Masters student at National Institute of Science Education and Research, Bhubaneswar-752050, email: diptanil.roy@niser.ac.in. A.V. Anil Kumar is with National Institute of Science Education and Research, Bhubaneswar-752050, email: anil@niser.ac.in