An Initial Condition Dependent Oscillation Control in an Odd and Even Number Van der Pol Ring.

Vinod V. and Bipin Balaram and M.D. Narayanan and Mihir Sen *

This paper addresses the spatial spread of control in a ring network based on change in initial conditions as well as the change in system parameters. The initial conditions needed for studying the periodic dynamics is done by implementing a generalised shooting strategy [1]. Odd and even number of oscillators which affects the symmetry in the network are tested under these initial conditions. In an environment of self excited oscillators in a ring, a perturbation does not stay localized to its next immediate neighbors [2]. The motion of each member in the ring is idealized as the self excited oscillation of a Van der Pol oscillator [3]. The governing equation of the model is given by

$$\ddot{x}_i + \mu({x_i}^2 - \lambda)\dot{x}_i + \omega_i^2 x_i = \sum_{p=1}^{n} \frac{\sigma_D}{2^{p-1}}(\dot{x}_{i-p} - 2\dot{x}_i + \dot{x}_{i+p}). \quad (1)$$

The R.H.S. of the model indicates the dissipative coupling force between the i^{th} oscillator and all others in the ring. Some of the control dynamics based on the initial condition are discussed in the following results. The implementation of the generalised shooting strategy in the ring model is detailed in [4].

Full Control in an Even Number Ring

In an even ring network $(n = 4)$ with all the oscillators having the same natural frequency, amplitude death occurs when the oscillators are subjected to an initial condition $[(C, 0), (-C, 0), (C, 0), (-C, 0)]$, where $C = x_i(0)$ is a positive integer. The time plot starting from a unit value of C initially forms antiphase clusters among the alternate oscillators. Later all of them tend to die completely at a time of approximately 200 s in Fig. 1.

Negative control in an Odd number Ring

If a fifth oscillator is introduced in the ring by giving an initial condition $(0, 0)$, initially it tends to act as a 'phantom' oscillator. In the time plot which is Fig. 2, the odd oscillator remains idle for a certain period of time and the remaining oscillators are under amplitude death. After a certain time of around 230 s, the odd oscillator begins to oscillate along with others with a common periodicity thereby attaining synchronization. The time sample at which it begins to oscillate along with others will be referred as waking time (t_w). The waking time of the odd oscillator possesses its own characteristics, one being its dependence on the nonlinearity of oscillators in the ring for a fixed value of coupling coefficient.

*Vinod V., M.D. Narayanan., Department of Mechanical Engineering, National Institute of Technology, Calicut, Kerala, INDIA.email:p100007me@nitc.ac.in, mdn@nitc.ac.in. Bipin Balaram, Department of Mechanical Engineering, Amrita Vishwa Vidyapeetham, Coimbatore, Tamilnadu 641 112,INDIA. email:bipinbalaram@gmail.com. Mihir Sen, Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, U.S.A. email:msen@nd.edu

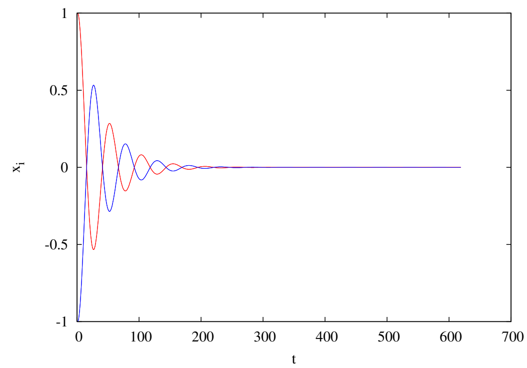

Figure 1: Full control based on initial condition.

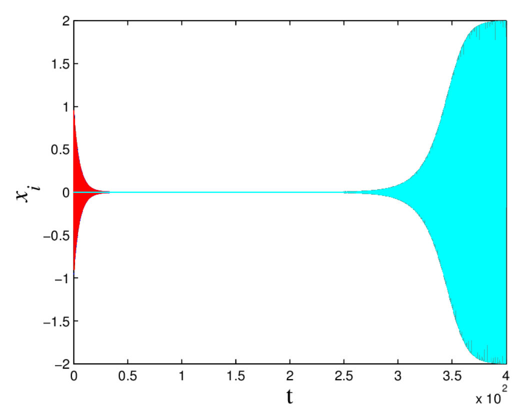

Figure 2: Waking of odd oscillator in the ring having $\mu = 0.1, \sigma_D = 0.1$.

Full Control of Oscillators with Frequency Detuning

Consider an even number ring under the initial condition $[(C, 0), (C, 0), (C, 0), (C, 0)]$ and having a frequency mismatch given to the first oscillator whose natural frequency is 80% of the remaining three oscillators. There exists a region 'A' in parameter space (μ, σ_D) of Fig. 3, where amplitude death happens in the time plot of all oscillators in the network.

Localised Control in the Ring

When an even number ring is subjected to the

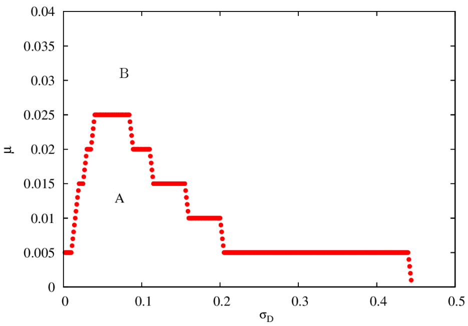

Figure 3: Amplitude death in nonidentical oscillators. A= Region of Amplitude Death, B= Region of No Death, for $\Delta\omega = 0.2$.

initial condition $[(C,0),(-C,0),(C,0),(-C,0)]$ or $[(C,0),(C,0),(C,0),(C,0)]$, the detuned oscillator along with its next immediate neighbors undergoes suppression in its dynamics. The oscillators which are not in direct contact with the detuned oscillator will not cause any change in amplitude, while the adjustment of phase and frequency occurs in the whole network when one moves along the axis of the coupling coefficient. All the oscillators are under suppression except third one (x_3) by comparing the displacement in the phase plane projection in Fig. 4.

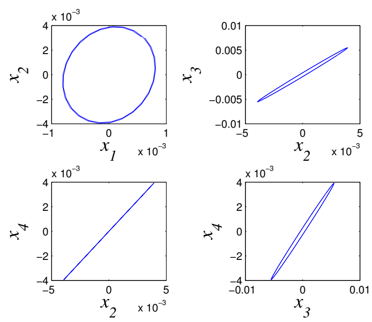

Figure 4: Localised control of x_1,x_2 and x_4.

Some of the results published in Vinod V. and Bipin Balaram and M.D. Narayanan and Mihir Sen, Effect of oscillator and initial condition differences in the dynamics of a ring of dissipative coupled van der Pol oscillators, *JMST, Springer*, **Vol. 29,5**, 2015, pp. 1931-1939.

References

[1] D. Li and J. Xu, *A new method to determine the periodic orbit of a nonlinear dynamic system and its period*, Engineering with Computers, 2005.

[2] M.A. Barron and M. Sen, *Dynamic behavior of a large ring of coupled self-excited oscillators*, Journal of Computational and Nonlinear Dynamics, 2013.

[3] M.A. Barron and M. Sen, *Synchronization of four coupled Van der Pol oscillators*, Journal of Nonlinear Dynamics, 2009.

[4] Vinod V. and Bipin Balaram and M.D. Narayanan and Mihir Sen, *Effect of number of oscillators on synchronization in a ring of coupled Van der Pol oscillators with local and nonlocal interactions.*, Journal of Nonlinear Dynamics, Under Review.