A Variant of the Mid-point Monodromy Algorithm

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Monodromy matrix based orbit search algorithms are useful for finding periodic orbits of dynamical systems. For 2 degrees of freedom Hamiltonian systems, convergence is usually achieved within 5 to 6 iteration with good accuracy [1] [2]. The algorithms are known to converge for both, stable as well as unstable orbits. Some modified algorithms which use an improved estimate of the time evolution part in the calculation either using mid-point, or Runge-Kutta [**?**] or a Taylor series [**?**] have also been proposed. However one major drawback of these algorithms is that often they miss a close-by orbit and converge to an orbit far away in the phase-space. The multi-step approach developed in [**?**] improves the accuracy of the time evolution part of the algorithm, whereas the major goal is to do evolution in either perturbation parameter or energy domain. A better and more precise evolution in these domains is expected to give a better result for the orbit search. In this paper we develop the formalism for the mid-point monodromy orbit in the energy domain.

We start with the iterative equation for increment in the configuration space variables around an initial orbit given by [2],

$$(1-\lambda_{N+1})Z_1 = \gamma_{N+1}$$

for an N point discretization of the orbit. Using Z_1 one arrives at the initial conditions of the next orbit. Compare the above equation with the one dimensional Newton-Raphson method where,

$$h = -\frac{f(a)}{f'(a)}$$

provides the increment in a to find zeros of $f(x)$ with initial guess being $x = a$. In case of the monodromy algorithms, the role of h is played by the four dimensional column vector Z_1. If the energy of the new orbit due to increments Z_1 happens to be $E_0 + \Delta$, the increment along the direction of Z_1 is chosen in such a way that the energy becomes $E_0 + \Delta/2$ instead. Just like the midpoint method the orbit at this energy is evolved once again to find a new matrix Λ_{N+1}. This matrix found at the midpoint is now used in the initial formula to solve for Z_1 and find the new increment. We compare our results for orbit accuracy using one mid-point step and two steps of half the sizes. A more detailed work for orbits search in the time domain as well as higher order calculations using Runge-Kutta like approach is in progress.

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