Separating a Heterogeneous Mixture of Chaotic Signals using Compressed Sensing

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1 Introduction

We consider the following problem. Given a heterogeneous (linear) mixture of chaotic signals corrupted with additive noise, how do we separate the individual signals? This problem arises in several contexts such as multiplexing of chaotic neural signals in the brain [1, 2], cryptography and steganography applications [3]. We briefly survey current methods of multiplexing of chaotic signals and show their inability to solve this problem. We then propose a novel solution using the paradigm of Compressed Sensing. Our approach can successfully separate a heterogeneous mixture of (say) 50 chaotic signals randomly chosen from a known set of $10,000$ signals (5000 logistic and 5000 tent map signals) in the presence of noise (see Figure 1).

2 Limitations of Existing Methods

Existing techniques to address the aforementioned problem can be broadly classified into two classes - (i) chaotic synchronization based methods [4]-[6], and (ii) symbolic sequence based methods [3, 7]. Both these classes of methods fail to solve the problem because of the following limitations. Chaotic synchronization methods work only for a homogeneous mixture of certain chaotic maps (and flows) under specific conditions only (for e.g., it fails for the Bernoulli shift map [6]). Furthermore, these methods are very sensitive to even small amounts of noise, round off errors, need a large number of iterates and fail to separate a large mixture of chaotic signals.

Though symbolic sequence based methods are immune to many of these defects, they have a serious limitation which is the difficulty of finite precision implementation for finding the initial condition from a symbolic sequence, a prerequisite for these methods to work. They also suffer from limited noise resistance and have not been tested for mixtures beyond 25 chaotic signals.

For instance, none of the existing methods can explain the demultiplexing of chaotic neuronal signals in the presence of interference from as many as $10,000$ neighboring neurons and neural noise, which the brain seems to be carrying out successfully. Recently, we have proposed such a scheme using Compressed Sensing [8].

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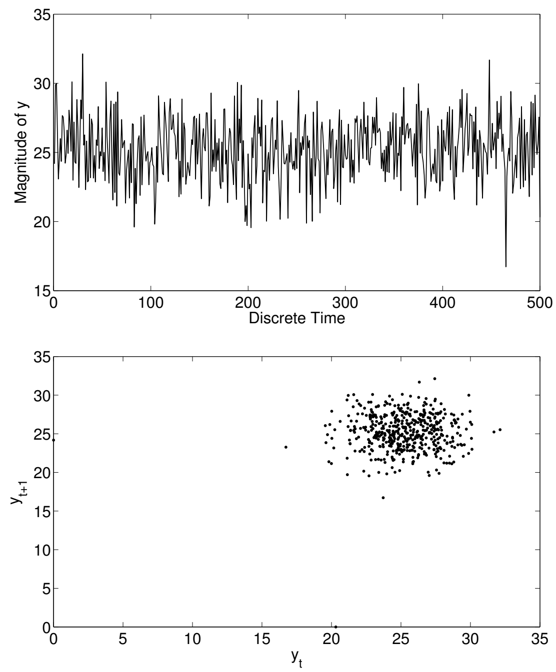

Figure 1: Top: Heterogeneously mixed signal (with additive noise) $y = (y_1, \ldots, y_t, \ldots, y_M)$. Bottom: A plot of y_{t+1} vs. y_t shows no discernible structure in the mixed signal, thereby making separation difficult.

3 A Novel Solution

We provide a novel solution to the problem of separating a heterogeneous (noisy) mixture of a number of chaotic signals randomly chosen from a large (known) set of signals using the paradigm of Compressed Sensing.

Compressed Sensing

Compressed Sensing (CS) is a signal detection framework [9] wherein if it is known that the signal being sensed is *sparse*, the number of measurements could be considerably reduced. We say a signal $x = (x_1, x_2, \ldots, x_N) \in \mathbb{R}^N$ is k-sparse if the number of nonzero entries in x is less than or equal to k. The measurement procedure is linear in nature and hence could be succinctly written as $b = Ax$, where A is an $M \times N$ sensing matrix and x is k-sparse. Observe that M corresponds to the number of measurements and N corresponds to the size of the signal being sensed.

Now, the problem is to estimate x given A and b. If

$M \geq N$, this is quite elementary. However, CS theory says that if x is k-sparse, to recover x from b, it suffices to have number of measurements $M \approx constant \cdot k \log \frac{N}{k}$ [10] and not as large as N. To recover x, there are many well-known algorithms which take the optimization approach (l_1 minimization [12]) or greedy approach (like Orthogonal Matching Pursuit [11]), provided A, the sensing matrix satisfies a property known as Restricted Isometry Property (RIP) [13]. For example, when the entries of A are all drawn independently from a Gaussian distribution, appropriately normalized, the matrix satisfies RIP. In [14], it was shown that a sensing matrix A constructed using chaotic signals satisfies RIP with overwhelming probability.

Another way to look at the aforementioned sensing problem is to think of $b \in \mathbb{R}^M$ as a *weighted linear combination* of some k (unknown) columns of A (duly addressing A as the *mixing matrix*), where the weights correspond to the nonzero entries of x. Then, solving for x tells us which columns participated in the mixing. This quite a useful view is what we will subscribe to in this paper.

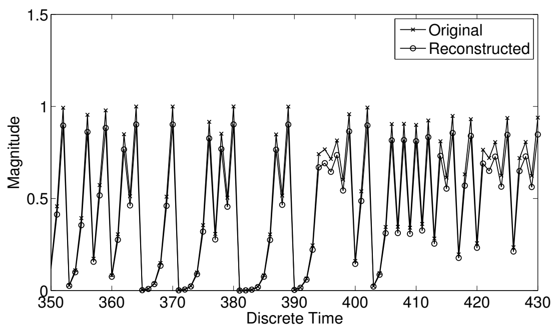

Figure 2: An example of a portion of an original and reconstructed chaotic signal using our proposed approach. Similar results were obtained for all 50 chaotic signals.

Proposed method

The multiplexed signal b is obtained by the weighted linear combination of k (or fewer) signals chosen from a collection of N chaotic signals (each of length M) drawn from logistic and tent maps. We view these N signals as the columns of the matrix A and the weights as the nonzero entries of x, which is k-sparse. Thus $b = Ax$. The signal b is then transmitted through a channel which adds some stochastic noise n (with mean 0, variance η) to it. The received signal now looks like $y = b + n = Ax + n$. Now, the problem is one of identifying the columns (chaotic signals from logistic map and tent map) in the mixing matrix A which participated in the heterogeneous mixture, given a noise corrupted version, y. This could be viewed as solving for x given the sensing matrix A and the (noisy) received signal y. One of the ways to estimate x in such a case is to solve the convex program

$$\min ||x||_{l_1} \qquad \text{subject to } ||Ax - y||_{l_2} \leq \eta,$$

which gives the sparse solution x^* [12]. The accuracy of the estimate x^*, however, depends on the variance η of the stochastic noise, n. This is characterized by [9]

$$||x - x^*||_{l_2} \leq C_1 \eta + C_2 . \frac{||x - x_K||_{l_1}}{\sqrt{k}},$$

where C_1, C_2 are constants and x_K is the same as x at the k leading entries and the rest set to zero. Figure 2 shows a simulation (with $M = 500$, $N = 10,000$ and $k = 50$) where all the mixed chaotic signals are recovered (almost perfectly) when $\eta = 0.1$.

4 Conclusion

The method that we have described could be readily extended for signals arising out of continuous-time chaotic dynamical systems as well. Although we demonstrated our method using two kinds of chaotic signals, a heterogeneous mixture of chaotic signals from both maps and flows (of multiple kinds) can be handled by the proposed method as long as the mixing matrix A satisfies the Restricted Isometry Property.

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