Chaotic Dynamics and Diffusion in a Piecewise Linear Equation

Pabel Shahrear, Leon Glass and Rod Edwards *

Genetic interactions are oftened modeled by logical networks in which time is discrete and all gene activity states update simultaneously. However, there is no synchronizing clock in organisms. An alternative model assumes that the logical network is preserved and plays a key role in driving the dynamics in piecewise nonlinear differential equations. We examine dynamics in a particular 4 dimensional equation of this class. In the equation two of the variables form a negative feedback loop that drives a second negative feedback loop. By modifying the original equations by eliminating exponential decay, we generate a modified system that is amenable to detailed analysis. In the modified system, we can determine determine in detail the Poincaré (return) map on a cross section to the flow. By analyzing the eigenvalues of the map for the different trajectories, we are able to show that except for a set of measure 0, the flow must necessarily have an eigenvalue greater than 1 and hence there is sensitive dependence to initial conditions. Further, there is an irregular oscillation whose amplitude is described by a diffusive process that is well-modeled by the Irwin-Hall distribution. There are a large class of other piecewise linear nonlinear networks that might be analyzed using similar methods. The analysis gives insight into possible origins of chaotic dynamics in periodically forced dynamical systems.

Full paper published by the AIP Publishing Pabel Shahrear, Leon Glass and Rod Edwards, Chaotic Dynamics and Diffusion in a Piecewise Linear Equation, *Chaos : An Interdisciplinary Journal of Nonlinear Science*, **25, 033103**, 2015.

References

[1] L. Glass and S. Kauffman *The logical analysis of continuous, non-linear biochemical control networks*,J. Theor. Biol. 39, 10329 (1973).

[2] L. Glass and J. S. Pasternack,, *Stable oscillations in mathematical models of biological control systems*, J. Math. Biol. 6, 207223 (1978).

[3] T. Geisel and J. Nierwetberg, Onset of diffusion and universal scaling in chaotic systems, *Phys. Rev. Lett.* , 48, 7 (1982).

[4] E. Plahte and S. Kjglum,, *Analysis and generic properties of gene regulatory networks with graded response functions*, Physica D 201, 150176 (2005).

[5] P. Shahrear, L. Glass, R. Wilds, and R. Edwards,, *Dynamics in piecewise linear and continuous models of complex switching networks*, Math. Comput. Simul., 110 (2015) 3339.

*Dr. Pabel Shahrear is an Associate Professor at the Department of Mathematics in Shahjalal University of Science and Technology, Sylhet, Bangladesh, email: pabelshahrear@yahoo.com. Professor Leon Glass is a faculty of the Department of Physiology, McGill University, Montreal,QC, Canada, email: glass@cnd.mcgill.ca, Professor Roderick Edwards is the chair of the Department of Mathematics and Statistics, University of Victoria, BC, Canada, email: edwards@uvic.ca