On the Effect of Nonlinear Perturbations on the Precession of Mercury's Orbital Perihelion

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Albert Einstein's original General Theory of Relativity in his series of papers between 1915-1917 proposed to explain the gravitational interaction in nature through the curvature of 4-dimensional spacetime. This involved expressing the fundamental processes as a series of nonlinear partial differential equations.

However the currently established process of General Relativistic calculations involves a linearization of the set of equations and a subsequent assumption of the existence of a linear solution. The essence of the curvature of spacetime is the metric $g^{\mu\nu}$ in which all the information is expressed. Currently popular methods of Linearized Gravity involves expressing any non-flat spacetime as:

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} + O((h^{\mu\nu})^2)$$

where $\eta^{\mu\nu}$ is the metric of the flat Minkowski spacetime [Signature: (-,+,+,+)] and $h^{\mu\nu}$ is the necessary first order perturbation.]

While this approach reduces the calculative complexity of the system, it in itself is not the correct approach to a completely accurate solution of a problem. This article seeks to address this issue by taking the nonlinear system of equations at face-value and trying to solve the resulting system using numerical simulations without greatly simplyfying it analytically by taking avoidable assumptions.

The line of research followed below takes a simple graviationally bound planetary system and seeks to solve the corressponding set of Einstein's Equation of General Relativity for that system:

$$\nabla^{\mu} G_{\mu\nu} = 0,$$

without assuming any linearizations. This involves solving a set of 16 equations.

System Under Consideration: The system used for this investigation is the Sun-Mercury pair in our Solar System. It is a bound-system and the appropriate metric used to describe the system is taken to be the most simple one, the Schwarzschild metric:

$$\begin{aligned} ds^2 &= -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1}dr^2 \\ &\quad + r^2 d\theta^2 + r^2 sin\theta^2 d\phi^2 \end{aligned}$$

The next line of analysis follows the general method of Linearized Gravity but without the usual assumptions except that of conservation of energy(classical result)angular momentum. This results in the elimination of two of the variables. The resulting equation is a differential equation of two variables which is directly solved using numerical methods and simulations. The result is slightly different from the linearized solution and the precession calculated for the perihelion is however not found to be a constant quantity on close inspection.

To Further explore this result two further avenues of research are followed:

- To use a different form of a metric, in this case Kerr, under appropriate conditions
- To vary the mass of the central body(in this case, Sun) to change magnitude of the spacetime curvature

The results from the above two situations give a detailed look into the exact solutions of Einstein's Equation and are an extension into the field of Geometrodynamics as a nonlinear study of spacetime.

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