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Stable multiple vortices in collisionally inhomogeneous attractive Bose-Einstein condensates

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We study the stability of solitary vortices in an ef- fectively two-dimensional trapped Bose-Einstein condensate (BEC) with a spatially localized region of the self- attraction. Solving the respective Bogoliubov-de Gennes equations and running direct simulations of the underly- ing Gross-Pitaevskii equation, we find that the vortices with topological charge up to $S = 6$ (at least) are stable above a critical value of the chemical potential (i.e., be- low a critical number of atoms, which increases with S). To the best of our knowledge, this is the first example of a setting which gives rise to stable higher-order vor- tices, $S > 1$, in a trapped self-attractive BEC. The same setting may be realized in nonlinear optics too.

The most natural setting for hosting vortices is pro- vided by a pancake-shaped BEC, which is strongly confined in one direction (z) and weakly confined in the transverse plane. Although stable vortices with topological charge $S = 1$ have been predicted in the effectively 2D trapped self-attractive BECs [1]-[3], higher-order vor- tices were found to be unstable in the same models. The present work aims to predict stable vortices with $S2$ in a condensate with spatially localized attractive non- linearity, which is known to support stable fundamen- tal solitons (with $S = 0$, but not vortices) in related models [4, 5]. To the best of our knowledge, the setting elaborated in the present work is the first system which supports stable higher-order vortices in the trapped self- attractive BEC.

$$i\hbar\frac{\partial\psi}{\partial t} = (-\frac{\hbar^2}{2m}\nabla^2_{3D} + \frac{m}{2}[\Omega_r^2(x^2+y^2) + \Omega_z^2 z^2] + \frac{4\pi\hbar^2 a^s N}{m}|\psi^2|)\psi \quad (1)$$

where 2 is the 3D Laplacian, m is the atomic mass, Ω_r and Ω_z are frequencies of the radial and axial confinement, as is the s-wave scattering length, which is negative/positive for the attractive/repulsive binary interatomic interactions, and N is the number of atoms in the condensate, the norm of the wave function being 1.Factorizing the wave function, as usual, and integrating Eq. (1) over direction z, the 3D GP equation can be reduced to the 2D form [8], $i\partial\psi/\partial t = [-(1/2)\nabla^2_{2D} + (1/2)\Omega_r^2 + gN|\psi^2|]\psi$, where (r, θ) are the polar coordinates in the (x, y) plane, and $2a_s\sqrt{2\pi m\Omega_z/\hbar}$ is the effective strength of the non linearity. The

2D wave function also obeys the unitary normalization condition, $\int\int |\psi(x,y,z)|^2 = 1$. We here assume that the atomic density is not too high, therefore the deviation of the nonlinearity in the 2D equation from the cubic form [9] may be neglected.

It was predicted in various forms theoretically [5] and demonstrated experimentally [10] that the use of the Fes- hbach resonance [11] controlled by nonuniform magnetic or optical fields makes it possible to engineer spatially in- homogeneous nonlinearity [5]. Accordingly, modified 2D GP equation is rewritten as

$$i\hbar\frac{\partial\psi}{\partial t} = (-\frac{1}{2}\nabla^2_{2D} + \frac{1}{2}\Omega^2_r(r^2) + g(r)N|\psi^2|)\psi \tag{2}$$

with the nonlinearity coefficient, g, made a function of the radial coordinate. We choose a Gaussian spatial-modulation profile for the localized self-attraction:

$$g(r) = g_0 \exp(-b^2r^2/2) \tag{3}$$

where $b1$ determines the radius of the nonlinearity bearing area. We concentrate on parameter values which adequately represent the generic case, $g_0 = 1, b = 1.5$ and $\Omega_r = 1$ (in fact, $\Omega_r \approx 1$ may be fixed by rescaling). Indeed, the interplay of the trapping potential and local- ized self-attraction may produce nontrivial results when the respective radii are of the same order, ($\Omega_r \sim b^2$) (the cases of $\Omega_r \gg b^2$ and $\Omega_r \ll b^2$ reduce to the settings studied in works [1, 2] and [4], respectively).

The same equations (2) and (3), with t replaced by the propagation distance, may be realized in optics as a spatial-domain propagation model in the bulk waveguide, with the linear confining structure provided by transverse modulation of the refractive index, and the localized re- gion of the Kerr nonlinearity [5]. The latter element can be provided by inhomogeneously doping the host material with resonant elements providing local enhancement of the nonlinearity [12].

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