Recurrence quantification analysis of the low Reynolds number flow dynamics past a flapping wing

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This paper investigates the transition in the flow dynamics past a flapping wing in low Reynolds number regime through recurrence plots (RP) and recurrence quantification analysis (RQA). Recently, the unsteady aerodynamics of flapping wings at low Reynolds number has been receiving substantial attention in relation to the efficient design of biologically inspired flapping wing micro air vehicles (MAVs) [1]. Two dimensional numerical simulations of flow over a rigid flapping wing with prescribed pitch and plunge kinematics are performed in the present work, utilizing a finite volume based incompressible Navier-Stokes solver. The present system is inherently nonlinear as the fluid nonlinearity in terms of large viscous separation plays a key role. A bifurcation analysis is carried out with the nondimensional plunge amplitude (h) as the control parameter which reveals interesting dynamics in the trailing edge wake pattern as well as in the time histories of the aerodynamic loads. A proper understanding of the qualitative change in the dynamical behaviour is crucial to decide an optimal operating regime of the MAVs to achieve maximum propulsive efficiency. Therefore, there is a need to probe into the transitions in the associated dynamics. The nonlinear time series analysis tools like recurrence plots and various measures obtained from recurrence quantification analysis, employed in this study, provide a better insight into the dynamical transition.

Recurrence is the most rudimentary attribute of any dynamical system. Recurrence plot (RP), a visualization technique introduced by Eckmann *et al.*[2] enables us to investigate the m-dimensional phase space trajectory through a two-dimensional representation of its recurrences revealing distance correlations in the time series. Mathematically, it is a graphical representation of a symmetric binary square matrix ($R_{i,j}$) constructed by a binary mapping based on the criteria for recurrence depicted below for various values of i and j (different time instances), where both axes are time axes. The recurrence matrix is given by Eq. (1) for a phase space with N points.

$$R_{i,j} = \Theta(\epsilon - ||x_i - x_j||) \quad i,j = 1,2,3..N. \tag{1}$$

In the above equation, x_i is a point on the 'm' dimensional phase space, Θ is the Heaviside step function, ϵ is a predefined threshold and $||\cdot||$ indicates the L_2 norm (Euclidean norm). An optimal value of ϵ, which denotes the degree of closeness or the neighborhood size, needs to be chosen. $R_{i,j}$ is zero if the distance between the two points x_i and x_j in the phase space is greater than ϵ, else it is equal to unity. The recurrence plot contains black and white points corresponding to ones and zeros of the recurrence matrix, respectively. For all RPs, the main diagonal is a black line since the distance of a particular point from itself is zero. The characteristics of RPs for different system dynamics have been discussed in detail by Marwan *et al.*[3]. The RP corresponding to a periodic signal is characterized by equally spaced lines parallel to the main diagonal, whereas unequally spaced lines parallel to the main diagonal in the RP designate the quasi-periodic dynamics. However, the RP for a chaotic system is characterized by short broken diagonal lines parallel to the main diagonal along with single isolated points. On the other hand, an uncorrelated random signal such as white noise is characterized by single isolated points in the RP.

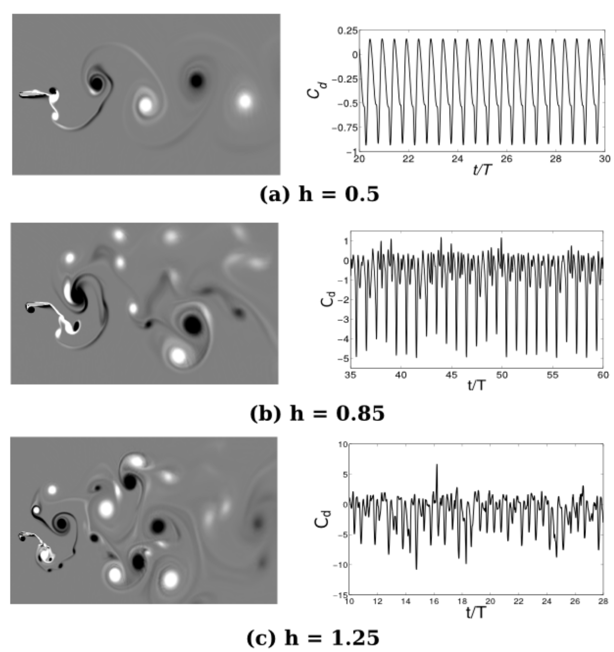

Figure 1: Vorticity contours and the corresponding time histories of coefficient of drag (C_d)

The vorticity contours and the corresponding time histories for the coefficient of drags (C_d) for three representative dynamics (at h = 0.5, 0.85 and 1.25) are presented in Figure 1. At h = 0.5, the trailing edge wake structures as well

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as the C_d time history is periodic (Figure 1(a)). With the increase in the bifurcation parameter, the flow periodicity is gradually lost and the switch in the dynamics can also be evidently observed in the corresponding C_d time history at h = 0.85 in Figure 1(b). With the further increase in the control parameter, the flow topology becomes completely chaotic and a chaotic time history of C_d is observed at h = 1.25 (Figure 1(c)). RPs are employed to characterize the different complex dynamical states in the chaotic transition of system dynamics. The periodic dynamics at h = 0.5 is also reflected on the corresponding RP with equally spaced parallel diagonal lines. The temporal transition at h = 0.85 in the dynamics is vividly seen in the corresponding RP in terms of different qualitative windows along the main diagonal line for different time intervals. At h = 1.25, the RP consists of very short broken lines parallel to the main diagonal and dots which characterizes the chaotic dynamics.

Recurrence quantification analysis (RQA) tools [4] are applied next, which give quantitative measures to complement the visual inspection of the recurrence plots. The RQA measures are computed for different nondimensional plunge amplitudes. In this paper, the following three RQA measures: recurrence rate (RR), determinism (DET) and length of the longest diagonal line (L_{max}) are investigated for different dynamics observed in the system. RR is the measure of the density of the recurrence points in a recurrence plot. DET is the ratio of the recurrence points forming the diagonal structures to all the recurrence points in the RP and L_{max} is the length of the longest diagonal found in a recurrence plot excluding the main diagonal. The RQA measures obtained from the aerodynamic load time histories provide a clear quantitative picture of the transition from the periodic to the chaotic regime. Since, the aerodynamic loads are representative of the flow field dynamics, the measures presented here also reflect the dynamics of the wake pattern as well. The transition regime is properly captured by the RQA measures and hence they can be used as precursors to identify the onset of aperiodicity and chaos.

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