Transport, Diffusion, and Energy in a volume preserving map

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We study the transport and diffusion properties of passive inertial particles described by a six-dimensional dissipative bailout embedding map. The base map chosen for the study is the three dimensional incompressible Arnold-Beltrami-Childress (ABC) map (Eq. 1) as a representation of volume preserving flows. There are two distinguishable cases: the two-action and the one-action cases, depending on whether two or one of the parameters (A, B, C) exceed 1 in the following modulo 2π map.

$$\begin{aligned} x_{n+1} &= x_n + A\sin(z_n) + C\cos(y_n) \\ y_{n+1} &= y_n + B\sin(x_{n+1}) + A\cos(z_n) \\ z_{n+1} &= z_n + C\sin(y_{n+1}) + B\cos(x_{n+1}) \end{aligned} \tag{1}$$

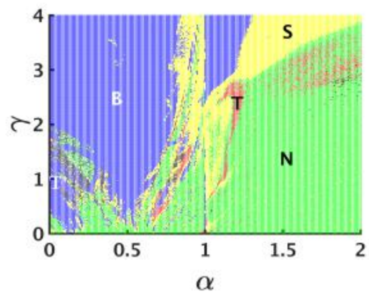

Figure 1: The phase diagram for the four diffusion regions in the embedding map for the two-action case for $(A, B, C) = (2, 1.5, 0.08)$ in the (α, γ) parameter space: the ballistic regime is marked in blue (B), the superdiffusive regime is marked in yellow (S), the normal diffusive regimes are marked in green (N), subdiffusion with stationary states is marked in red (T), and subdiffusion with nonstationary states is shown in black (T*). The map has been computed for 10 000 iterations averaged over 200 trajectories on a grid of 0.01×0.01 in the parameter space discarding 1000 iterates as transients.

The dynamics is governed by parameters (α, γ) which quantify the mass density ratio and dissipation respectively. There are important differences between the aerosol $(\alpha < 1)$ and the bubble $(\alpha > 1)$ regimes. We have studied the diffusive behavior of the system and constructed the phase diagram (Fig. 1) in the parameter space by computing diffusion exponents η. Three classes have been broadly classified – subdiffusive transport $(\eta < 1)$, normal diffusion $(\eta \approx 1)$, and superdiffusion $(\eta > 1)$ with $\eta \approx 2$ referred to as the ballistic regime. Correlating the diffusive phase diagram with the phase diagram for dynamical regimes seen earlier [1], we find that the hyerpchaotic bubble regime is largely correlated with normal and superdiffusive behavior. In contrast, in the aerosol regime, ballistic superdiffusion is seen in regions which largely show periodic dynamical behaviors whereas subdiffusive behavior is seen in both periodic and chaotic regimes. The probability distributions of the diffusion exponents show power law scaling for both aerosol and bubbles in the superdiffusive regimes.

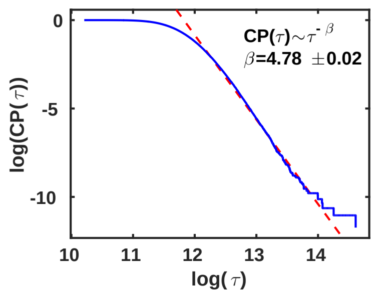

Figure 2: Cumulative recurrence time distribution for $(\alpha, \gamma) = (0.9, 1)$ in the chaotic regime for the two action case $(A, B, C) = (2, 1.5, 0.08)$. The recurrences have been computed for 10^7 iterations with 1000 discarded as transients.

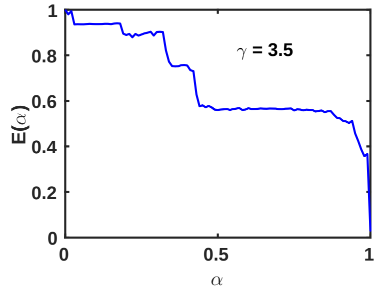

Figure 3: Average particle kinetic energies versus the mass density ratio for the two action case exhibit a Devil's staircase-like structure at $\gamma = 3.5$. The energy values have been computed for 10 000 iterations and averaged over 200 particles.

We further study the Poincáre recurrence times statistics of the system. Here, we find that recurrence time distributions show power law regimes (Fig. 2) due to the existence of partial barriers to transport. Moreover, the plot of average particle kinetic energies versus the mass density ratio for the two action case exhibits a Devil's staircase-like structure (Fig. 3) for higher dissipation values. We explain these results and discuss their implications for realistic systems.

References

[1] Swetamber Das and Neelima Gupte. Dynamics of impurities in a three-dimensional volume-preserving map. Phys. Rev. E, 90:012906, 2014.