Spacing distribution of localized states and their nearest neighbours in quantum chaos

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The study of level spacing distributions in quantum chaos has been inextricably linked with results from Random Matrix Theory(RMT). The BGS Conjecture states that the spectra quantum systems, whose classical versions exhibit chaos, have a spacing distribution which matches that of the Gaussian Orthogonal Ensemble(GOE), well-studied in RMT [1]. Deviations from these results can be attributed to quantum phenomena like localization and dynamical tunneling [2]-[5].

Some quantum chaotic systems contain a class of localized states, called 'scarred' states, which are quantum manifestations of unstable periodic orbits in the classical counterparts of these systems. The wave functions are localized in the vicinity of these periodic orbits, leading to the so-called scars. This phenomenon has been well-studied in various systems [6][7].

However, the spacing distribution between energy levels corresponding to scarred states and their nearest neighbours that (usually) correspond to chaotic states has not been studied so far. We propose a one-parameter RMT model with the parameter μ representative of the coupling between localized states and their nearest neighbours.

This 3×3 model can reproduce the distribution of the ratio of level spacings between a localized state and its chaotic or 'random' neighbours. This ratio is defined as follows: If E_n is the n-th eigenvalue of the system, then the spacing between nearest neighbours is defined as $s_n = E_{n+1} - E_n$. From this, the ratio of spacings, $r_n = s_{n+1}/s_n$ is calculated and its distribution $P(r)$ is obtained. From this, the Wigner surmise can be derived [8], and it takes the form:

$$P_W(r) = \frac{8}{27}\frac{(r+r^2)}{(1+r+r^2)^{5/2}} \tag{1}$$

This quantity (the distribution of ratios) is used as it does not require unfolding of the spectrum.

The applicability of this model has been demonstrated for quantum chaotic systems like coupled quartic oscillators and quantum billiards. In these models, the localized states are identified using information entropy. From this, the spacing between energy levels corresponding to a localized state and its nearest neighbour is calculated and the distribution of the ratio of the spacings is determined. The value of the model parameter μ corresponding to this distribution is then found.

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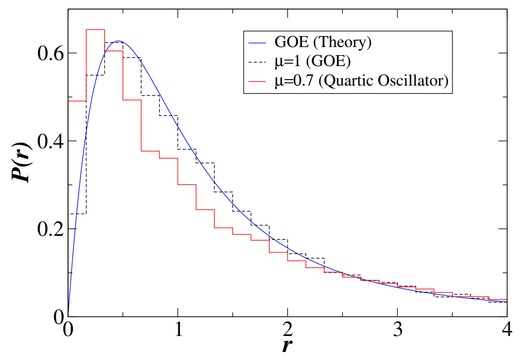

Figure 1: The blue curve denotes the formula for distribution of the ratio of consecutive level spacings *P(r)* for GOE. The broken line is the histogram obtained by numerical fitting for GOE from the 3×3 model, with the value of the parameter μ=1. The value μ=0.7 obtained from the model corresponds to the distribution of the ratio of spacings between a localized state and its nearest neighbours for the quartic oscillator.

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