Self-similar fractal spectrum in quantum mechanical systems

Jayendra N. Bandyopadhyay, Rashmi J. Sharma, and Tapomoy Guha Sarkar *

Fractal spectrum in quantum mechanical system was first observed in Harper model [1]. This model describes a very well known condensed matter system of electrons moving on a two-dimensional plane under the influence of a transverse magnetic field. This fractal spectrum is known as Hofstadter butterfly [2]. Later, similar butterfly fractal spectrum was observed in periodically kicked Harper model [3], and also recently in on-resonance double-kicked rotor model [4]. More recently, a different looking butterfly spectrum is observed in a double-kicked $SU(2)$ system, that is the double-kicked top system [5, 6].

Most of the previous studies focussed only on the fractal properties of the spectrum. The study of the universal self-similar behavior of the spectrum is completely missing in the literature. For example, the Hofstadter butterfly spectrum was only studied for a given value of the magnetic flux. However, this butterfly spectrum clearly shows self-similar nested set of butterflies over a range of values of the magnetic flux. Similarly, for the double kicked top model, the spectrum is only studied for a fixed value of a parameter; whereas, the spectrum clearly shows a set of butterflies in recursive manner.

Only very recently, a study has explored the self-similar properties of the Hofstadter butterfly. Following this line, we have studied self-similar properties of the fractal spectrum of the double-kicked top system. Even though, fundamentally the Hofstadter butterfly and the butterfly spectrum of the double kicked top system are different, but they share identical self-similar properties. This talk will discuss about self-similar properties of different quantum fractal and its relations with other geometric fractals.

Relevant publication: This talk will be based on a recent submission (also available in [7]) and some of the recent unpublished results.

References

[1] P. G. Harper, Single band motion of conduction electrons in a uniform magnetic field. *Proc. Phys. Soc. A*, 68:874, 1955.

[2] D. R. Hofstadter, Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields. *Phys. Rev. B*, 14:2239, 1976.

[3] R. Ketzmerick, G. Petschel, and T. Geisel, Slow decay of temporal correlations in quantum systems with Cantor spectra. *Phys. Rev. Lett*, 69:695, 1992.

[4] J. Wang and J. Gong, Proposal of a cold-atom realization of quantum maps with Hofstadters butterfly spectrum. *Phys. Rev. A*, 77: 031405(R), 2008.

[5] J. Wang and J. Gong, Butterfly Floquet Spectrum in Driven SU(2) Systems. *Phys. Rev. Lett.*, 102:244102, 2009.

[6] J. N Bandyopadhyay, J. Wang, and J. Gong, Generating a fractal butterfly Floquet spectrum in a class of driven SU(2) systems: Eigenstate statistics. *Phys. Rev. E*, 81:066212, 2010.

[7] R. J. Sharma, J. N Bandyopadhyay, and T. Guha Sarkar, Self-similar spectrum in effective time independent Hamiltonians for kicked systems. *arXiv:1504.06090*.

*Jayendra N. Bandyopadhyay is with Dept. of Physics, BITS-Pilani, Pilani 333031, email: jnbandyo@gmail.com. Rashmi J. Sharma is with Dept. of Physics, BITS-Pilani, Pilani 333031, email: jangid.rashmi@gmail.com. Tapomoy Guha Sarkar is with Dept. of Physics, BITS-Pilani, Pilani 333031, email: tapomoy1@gmail.com.