Entanglement in two-body random ensembles

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The entanglement measures, introduced in the context of Quantum Information Science, are used to characterize complexity in quantum many-body systems. Entanglement and delocalization are found to be strongly correlated for disordered spin-1/2 lattice systems [1]. The entanglement between two spatial partitions of a system is now greatly-appreciated as a useful characterization of quantum correlations in many-body wave functions. In the present work, by employing one- plus two-body random matrix ensembles for fermions [EGOE(1+2)]as well as for bosons[BEGOE(1+2); 'B' for boson], we study the bipartite entanglement measure - namely, the reduced von–Neumann entropy (the entanglement entropy).

Embedded Gaussian orthogonal ensemble (EGOE) of random matrices (for time-reversal and rotationally invariant systems) with one plus two-body interactions for fermions (bosons), introduced in the past, are paradigmatic models to study the dynamical transition from integrability to chaos in finite interacting many-body quantum systems [2]. It is important to note that EGOE are generic, though analytically difficult to deal with, compared to lattice spin models as the latter are associated with spatial coordinates (nearest and next-nearest neighbor interactions) only. Moreover, it is seen that the universal properties derived using EGOE apply to systems represented by lattice spin models. EGOE(1+2), as a function of the two-body interaction strength λ (measured in units of the average spacing between the one-body mean-field sp levels), exhibit three transition or chaos markers $(\lambda_c, \lambda_F, \lambda_t)$: (a) as the two-body interaction is turned on, level fluctuations exhibit a transition from Poisson to GOE at $\lambda = \lambda_c$; (b) with further increase in λ, the strength functions (also known as the local density of states) make a transition from Breit-Wigner (BW) form to Gaussian form at $\lambda = \lambda_F > \lambda_c$; and (c) beyond $\lambda = \lambda_F$, there is a region of thermalization around $\lambda = \lambda_t$ where basis dependent thermodynamic quantities like entropy behave alike.

The bipartite entanglement entropy (S^{EE}) is commonly used to study the strength of the quantum correlations between two parts of a many-body system. Given a partition of the system into parts A and B, the S^{EE}between A and B in the eigenstate with energy E, $|\psi_E\rangle$ is given by, $S^{EE}(E) = -Tr\rho_A(E)\log\rho_A(E) = -\sum_\mu \tau_\mu(E)\log\tau_\mu(E)$. Where $\tau_\mu(E)$ are the eigenvalues of the reduced density matrix $\rho_A(E) = Tr_B\rho(E)$ of the A part, obtained from the full density matrix $\rho(E) = |\psi_E\rangle\langle\psi_E|$ of the eigenstate by tracing out B degrees of freedom. Here, we have studied the bipartite entanglement entropy (S^{EE}) for m-interacting fermions (bosons), interacting via two-body interaction, in a space of N single particle states. the S^{EE} is obtained between partitions of equal halves of the single particle states. The system Hamiltonian H is given by $H = h(1) + \lambda V(2)$, where $h(1)$ represents the mean field part (1-body part) of the interaction, described by fixed single particle energies and $V(2)$ is two-body part described by matrix elements V_{ijkl}, which are treated as gaussian random variates; λ is the parameter to control the effective strength of the two-body part of the interaction. Here, we analyze the entanglement entropy for fermion (boson) ensembles using all the eigenstates. The results are shown in the figure 1. Also participation ratio defined as PR$(E) = \{d\sum_k |C_k^E|^4\}^{-1}$ is studied and its correlations with entanglement entropy are analyzed. The results are consistent with those obtained in [3] where Bose-Hubaard model and spin models have been employed.

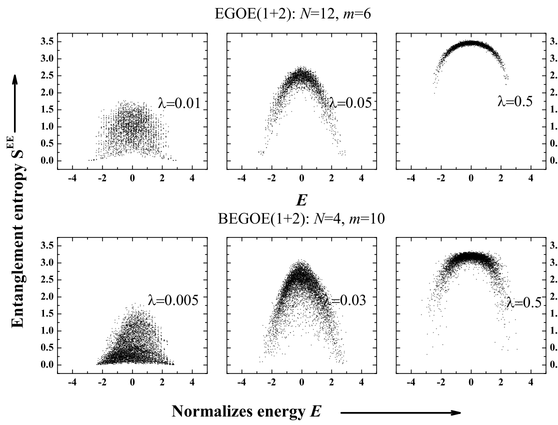

Figure 1: Entanglement Entropy (S^{EE}) as a function of normalized energy E, of the eigenstates for EGOE(1+2) ensemble with 6 fermions in 12 single particle states and for BEGOE(1+2) ensemble with 10 bosons in 4 single particle states for different values of interaction strength, λ.

The part of work presented here is carried out in collaboration with V. K. B. Kota.

References

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