Data assimilation in chaotic Chua circuit

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Data assimilation (DA) is a tool that improves forecast of a future state of a dynamical model system by taking into account the noisy observational data. Noisy observations are incorporated into the numerical model and the most probable forecast is given [1, 2]. This tool is indispensable in many fields of earth sciences, particularly in weather forecasting and oceanography. Quite a few methods for DA have been developed [3, 4, 5]. We will look at two methods of sequential DA for discrete data sets.

When two or more chaotic systems are coupled under certain conditions, the trajectories may eventually get synchronized. There are many types and definitions of synchronization of chaotic systems in literature [6, 7]. However, for our purposes we will look at identical chaotic dynamical systems coupled at discrete time instances. This can be referred to as sporadic or impulsive synchronization. Synchronization of the numerical model with observations by means of unidirectional coupling can be thought of as a method of DA.

Ensemble Kalman filter (EnKF) is a very popular method of DA. It is a Monte-Carlo approximation of the Kalman Filter (KF). At each discrete time instance, a random sample of points or ”ensemble” is propagated according to the theoretical model and the true mean and covariance as in KF are replaced by the sample mean and covariance respectively.

Our main aim in this paper is to compare the dependence of the two aforementioned methods of DA on parameters like observational noise, time period between two subsequent observations, and time duration for which the system is observed. It has been observed that the aforementioned factors play an important role in assimilation, therefore one would naturally like to know how different DA methods compare on the basis of these factors.

The model we used for these studies is the low dimensional Chua's circuit. In this paper, we first introduce the basic model and it's dynamics and go on to briefly describe DA methods of synchronization and EnKF. We describe results comparing the EnKF and synchronization using synthetic data as well as experimental data.

Full paper to be submitted.

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